

Population Projection using Mathematical Curves: Will Indian Population Stabilises by 2050?

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Abstract

The main objective of this study is to develop a simple mathematical approach to predict the population of India up to the end of present century, without attempting any justification of particular assumptions about fertility, mortality and migration, like cohort component approach. In this study the most popular curve used for projection of population i.e. logistic curve is used as well as another Gaussian curve is used to predict the population of India utilizing decadal population from 1901 to 2001. A comparison between outputs of the two curves is indicating that the Gaussian curve is more reasonable than Logistic curve. According to the Gaussian curve population of India will established nearby 2051 however it is 2021 in case of logistic curve that is not reasonable. Also population projection of male and female is obtained separately which shows that female population will be more than male in future perhaps due to the higher life expectancy of female.

Introduction

A population model is a type of mathematical model that can be applied to the study of population dynamics. Modelling of dynamic interactions in nature can provide a manageable way of understanding how numbers change over time. There are various methods to develop a population growth model. Population Growth has become one of the most important issues in the world (Ehrlich & Holdren, 1971). The idea of the future population is achieved with the help of projection. The demand of precise projected figures is always requisite for government organizations, actuaries, program makers for better planning of well beings of human population. The need of population projection arise for future planning regarding well being of population under the constraints of natural resources, infrastructure of the country to control the consequences of population explosion. Population change influences, and is influenced by human activity as well as earth's environments.

Population sizes and growth in a country directly influence the condition of economy, policy, culture and education of the country. Keyfitz (1972), Lutz (2013) and Repetto (1987) worked on the issue of population projection. Population Projection is currently produced by many organizations, including International, national and local governments as well as private companies. In India the Registrar General of India (RGI) is the main organization which produced population projection for all states including country. The method used by RGI, (1997, 2006) does not yield an assessment of uncertainty about future population generally. The same phenomenon is also observed in other projections like population foundation of India and population reference bureau (2007) and Dyson (2004).

Population projection is an extrapolating exercise of future on the basis of past trends. Population projection methods enables updating the population data with significant degrees of approximation to reality. Based on assumptions scientifically supported, it is possible to have estimates of size and composition of populations, with disaggregation responding to the interest of researchers and users of information. Population projections, often made by government agencies for planning purposes, is treated as a useful data source for emergency management planning, marked Ferrier (1999). The present and past population record can be obtained from the census population records. After collecting these population figures, the population is predicted using various methods as suitable for that country, considering the growth pattern followed by the country.

Smith and Sincich (1992) favours the use of simple exponential extrapolation, Long (1995) for full cohort-component projections and Ahlburg (1995) for more complex methods involving economic-demographic models. The trend based methods assume that population growth follows natural laws and, therefore, can be expressed in mathematical or graphical form. A mathematical model or expression is a set of formulas or equations based on quantitative description or real world experience and formed in the hope that the predicted behaviour will resemble the real behaviour on which it is based (Glenn Ledder, 2005).

Basically, population is forecasted by examining and projecting past trends into the future. Various types of expressions have been used such as linear, geometric, exponential, logarithmic, etc., to explain past historical growth and predict future growth. Graphic projections are most commonly made using arithmetic, semi log, or probability paper. The data used in the plotting are historic data from decennial census reports and from available local or State census reports from intermediate years. The historic data are often plotted on all three types of graph paper, and the plot which comes closest to a straight line indicates the mathematical form to be used for the projection. In using the plotted information for projection purposes, we assume that the condition implied by the straight line will continue into the future. Arithmetic plots for India have shown that at first the population increased at a low rate, then accelerated for a period of time, and later, as the countries matures, the rate of growth decreased. When the data is plotted on arithmetic paper, the result comes in the form of typical “S” shape curve, which is known as the logistic curve.

Mathematical curves are used when the growth rate of population due to births, deaths and migrations takes place under normal situation and it is not subjected to any extraordinary changes like epidemic, war, earth quake or any natural disaster. If the population of a country is plotted with respect to time, the curve so obtained under normal condition looks like S-shaped curve called as logistic curve. Various forms of logistic curves were used by many researches for population projection of the different countries. Rahul et al. (2007) used this curve under Bayesian setup to predict population of India and some states. In the case of estimating Indian population or population of its sub-regions, it is revealed that logistic model grossly underestimates true population (Gupta et al. 2012). In the effort to overcome this problem we have tried to fit Gaussian curve as an alternative of the logistics curve. It projects more realistic values and provides better projection for future as compared to logistic curve. The projection shows Indian population is approaching to an era of stability but its longevity is not sure. Present study shows that after reaching a maximum level of population around 2050, it will start decline.

Data & Methodology

In the present paper four parameter logistic and Gaussian curves have been used due to more flexibility. The parameters involved in the functions are estimated by least square method. We have used the data of census of India (from 1901 to 2001; 11 time points) and forecast the total population as well as the male and female population of India up to the end of present century.

Year	1901	1911	1921	1931	1941	1951	1961	1971	1981	1991	2001
Total	238.4	252.1	251.3	279.0	318.7	361.1	439.2	548.2	683.3	846.4	1027
Male	120.9	128.3	128.5	143.1	163.8	185.5	226.2	284.0	353.3	440.6	532.2
Female	117.5	123.7	122.7	136.0	154.8	175.5	213.0	264.1	330.0	405.8	496.5

Logistic Population Growth

The logistic growth model was first used by Verhulst in 1845 and originated from observations of biological reproduction process. Population growth rate is measured in number of individuals in a population (N) over time (t). The term for population growth rate is written as (dN/dt) . The equation of four parameter logistic curve is given below:

$$y = a + \frac{4bn}{(1+n)^2} \quad \text{where } n = e^{\left(\frac{-(x-c)}{d}\right)}$$

and the estimates of the parameters are, $a=220.5396$, $b=1017.4684$, $c=2022.6931$ and $d=21.9860$.

Gaussian Population Growth

The Gaussian distribution is very important in practice, many distributions resembles Gaussians, and the Gaussian distribution is relatively easy to work with – can be used to estimate uncertainties, etc. Here we have used Gaussian four parameter curve for high precision with the following equation.

$$y = a + be^{\left(-0.5\left(\frac{x-c}{d}\right)^2\right)}$$

and the estimates of the parameters are, a=234.3968, b=1347.0176, c=2046.4564 and d=44.1450.

Table 1: Projection of India's Population using Logistic and Gaussian growth models

Year (x)	Population (y)	Logistic Population Projection			Gaussian Population Projection		
		Predicted population	95% upper CI	95% lower CI	Predicted population	95% upper CI	95% lower CI
1901	238.4	236.47	228.63	244.32	240.31	233.30	247.32
1911	252.1	245.54	238.89	252.18	246.55	240.69	252.42
1921	251.3	259.65	254.13	265.17	258.14	253.35	262.94
1931	279.0	281.50	276.49	286.50	278.46	273.92	282.99
1941	318.7	314.94	309.45	320.44	312.05	306.85	317.25
1951	361.1	365.32	358.91	371.74	364.43	358.64	370.22
1961	439.2	439.30	432.54	446.06	441.24	435.59	446.90
1971	548.2	543.72	537.14	550.30	546.96	541.30	552.63
1981	683.3	682.42	674.52	690.31	683.10	676.22	689.97
1991	846.4	850.20	841.10	859.29	846.31	839.19	853.43
2001	1027.0	1025.64	1013.53	1037.75	1027.14	1016.85	1037.43
2011		1169.32	1123.62	1215.01	1210.04	1177.30	1242.79
2021		1236.50	1128.36	1344.64	1375.08	1298.16	1452.00
2031		1202.54	1022.83	1382.26	1501.33	1358.24	1644.42
2041		1080.19	848.51	1311.87	1571.16	1343.99	1798.34
2051		910.39	664.22	1156.56	1574.30	1254.83	1893.77
2061		736.65	510.62	962.67	1510.26	1104.02	1916.50
2071		586.85	400.56	773.14	1388.52	915.52	1861.51
2081		470.95	329.06	612.84	1226.17	717.76	1734.58
2091		387.37	285.46	489.29	1044.03	536.69	1551.37

* The values relate to populations in millions.

Above comparison between projected populations by both the curves meets (more or less) the actual population up to 2001 but as we proceed logistic curve started gradual underestimation which can be noted at a real time point of 2011 (i.e. 1169.32 million by logistic projection but actual 1210.1 million population by census 2011) and similarly goes with more decreasing rate of increase in population but it is unacceptable because each population has its own momentum. It should be noted that, a population typically does not stop growing or declining the instant its fertility reaches replacement level (Espenshade et al. 2015). Momentum can play a large or a small role in population dynamics (Bongaarts, 1994, 2007). So, it will take time to be stabilized as far as Indian population is concerned but not as early as this projection shows i.e. by 2021. Also, at the end of the projection i.e. by 2091 Logistic curve, predicts the same population as it was in 1950s in India, which is sharp decline, it is not convincing factor in the usual condition.

Our projection by Gaussian curve up to 2001 it is more or less follows the pattern of population with few degree of variation. And this projection meets population at the time point 2011 far better than Logistic. It also projects a gradual progress and decline over the time. This projection of population also portrays a picture containing the concept of momentum. Stability in Indian population according to Gaussian curve will be around 2050 when the projected population reached at 1574.30 although India will have reached replacement level fertility earlier. After stability it shows a decline same as other developed countries who have been suffering from shrinking population. And become as it was in the starting of the century i.e. from 1574.30 by 2051 to 1044.03 by 2091.

The above Fig1 shows that the stability attained by the population of India in 2021 and further the decline in the population started in 2031 which seems to be unrealistic. In customary condition this forecast is not going to be attained. In Fig 2, curve attains its peak by 2051 which is the

stability status for the population. And at the end of the century more or less the population will be same as it was at the beginning. Indeed, the inability to determine a set of components in accord with their known long-term trends, the logistic curve model would presumably be an indication of a discontinuity and the establishment of a new logistic curve. So that here Gaussian curve is tested and found to be a good substitute for logistic and other complex methods.

Here, in this paper we have tried to fit Gaussian curve on the data of census for population projection of India. This examination of the Gaussian curve has shown that it may be fitted to population data covering extended periods of growth and provide an acceptable model of that growth. It has further been shown that for India a period of stability has occurred in 2051 and decline in population started in 2061, which is more acceptable than what is given by the logistic curve.

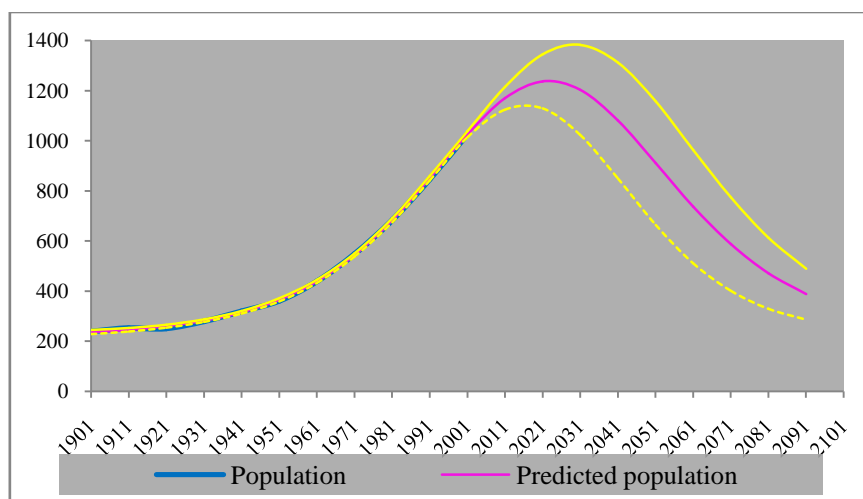


Fig 1: Logistic curve of population projection

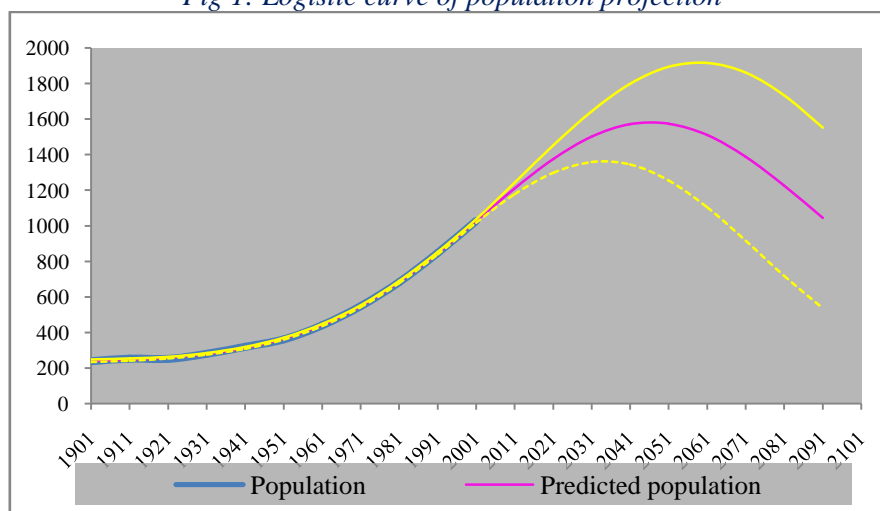


Fig 2: Gaussian curve for population projection

Table 2: Comparison in various population projections

Year	Projection by UNDP**	Projection by World Bank	Projection by MoH&FW	Projection by Tim Dyson (2004)	Projection by Rahul et al. (2007)	Projection by Gaussian	Projection by Logistic
2001	1071.4	1071.4	1028.6		1026	1027.1	1025.6
2011	1247.2	1247.2	1192.5	1204	1197	1210.0	1169.3
2021	1383.1	1397.4	1339.7	1362	1345	1375.1	1236.5
2031	1512.9	1524.1	NA	1465	1463	1501.3	1202.5

2041	1605.3	1612.3	NA	1543	1549	1571.2	1080.2
2051	1658.9	1658.9	NA	1579	1593	1574.3	910.4
2061	1678.5	NA	NA	NA	NA	1510.3	736.6
2071	1665.1	NA	NA	NA	NA	1388.5	586.8
2081	1626.9	NA	NA	NA	NA	1226.2	470.9
2091	1574.7	NA	NA	NA	NA	1044.0	387.4

** Projection by UNDP is done in 2020, 2030, 2040.....2090.

Adopted method is based on simple assumptions and a very simple procedure compared to other projections methods i.e. Bayesian setup, cohort component method, Gompertz model etc. A very less complex mathematical procedure is adopted and it provides more or less the same figure as other projection. For 2011, our projection i.e. 1210 is most suitable in comparison to other again for 2021 and 2051 it is 1375.1 and 1574.3 respectively, almost same to projection of Tim Dyson (2004) and also to other. Hence, we can have a easier method i.e. Gaussian curve method for population projection. A comprehensive comparison with our projection is given in the above table.

Overall population will certainly decrease in the process of time because the Total fertility rate (TFR) of India is decreasing according to various reports as NFHS, SRS and census. Most of Indian states especially southern states have already achieved replacement level fertility in numbers 24 out of 36 states & UT's. The data of NFHS-4 depicts the declining TFR and shows the attainment of replacement level of fertility by states of India. In NFHS-4, TFR declined to 2.2 children per woman from 2.7 in NFHS-3. Thus, it is moving closer to target level of 2.1. But still some major populated states have to achieve replacement level fertility.

These states have rapid declining TFR. For example, Uttar Pradesh was having 3.8 TFR in 2005-06 (NFHS-3) but it rapidly declined by 1.1 in a decade as it is now 2.7 in 2015-16 (NFHS-4). We can draw an assertion that in near future circumstances will lead the population to make it stable. After stability it will start declining. The above growth curves are applied separately to get a clearer picture of male and female population. Expanding the above methods for male and female separately some interesting results have been observed.

Table 3: Projection for male population of India by Logistic and Gaussian curve

Year (x)	Population (y)	Logistic curve			Gaussian curve		
		Predicted population	95% Lower CI	95% Upper CI	Predicted population	95% Lower CI	95% Upper CI
1901	120.9	120.47	116.63	124.30	122.43	118.50	126.36
1911	128.3	125.17	121.91	128.42	125.66	122.37	128.96
1921	128.5	132.51	129.80	135.22	131.70	129.00	134.39
1931	143.1	143.90	141.44	146.35	142.31	139.76	144.85
1941	163.8	161.38	158.69	164.07	159.91	156.99	162.83
1951	185.5	187.77	184.63	190.92	187.38	184.13	190.64
1961	226.2	226.58	223.26	229.90	227.69	224.51	230.87
1971	284.0	281.37	278.14	284.60	283.10	279.92	286.29
1981	353.3	354.01	350.14	357.88	354.27	350.41	358.14
1991	440.6	441.43	436.97	445.88	439.23	435.23	443.22
2001	532.2	531.82	525.89	537.76	532.72	526.95	538.50
2011		604.07	581.87	626.26	626.35	608.05	644.65
2021		635.00	583.06	686.95	709.51	666.74	752.28
2031		613.08	527.82	698.33	771.33	692.22	850.43
2041		546.58	438.11	655.05	802.94	678.17	927.70
2051		457.64	343.82	571.46	799.57	625.43	973.72
2061		368.55	265.21	471.89	761.76	542.17	981.34
2071		292.88	208.54	377.22	695.14	441.87	948.42
2081		234.99	171.30	298.68	609.15	339.72	878.58
2091		193.60	148.23	238.98	514.79	248.96	780.61

* The values relate to populations in millions.

According to logistic curve, starting from 1901 with gradual growth rate then accelerated after mid nineteenth century from 187.77 by 1951 to 635.00 by 2021 and gets stability point after attaining stability nearby 2021 it will decline with increasing rate of decrease i.e. from 635.00 by 2021 to 193.60 by 2091. In the same way, according to Gaussian curve male population is also starts gradual increase then changes rapidly up to saturation point i.e. 692.22 by 2031 and then decline gradually up to 514.79 by 2091. After 2001 Gaussian curve projects with more precision than logistic curve. Stabilization in male population according to logistic curve will occur around 2021 whereas according to Gaussian curve it will around 2041. This provides a clue for stabilization of total population around 2050. Logistic curve projects rapid decline in population and at the end of century i.e. around 2090 it shows population will be almost same as it was in 1950s. Whereas Gaussian curve projection meets almost same male population as it is in starting of the century i.e. by 2001. After attaining stability, the gap between logistic projection and Gaussian projection is widening with increasing rate. Its lower variant of Gaussian is equal to the upper variant of logistic curve.

Comparing these projections of male population with other reports like Youth in India-2017, World Bank projection is almost same. Compatibility of Gaussian curve for projection is found to be most suitable.

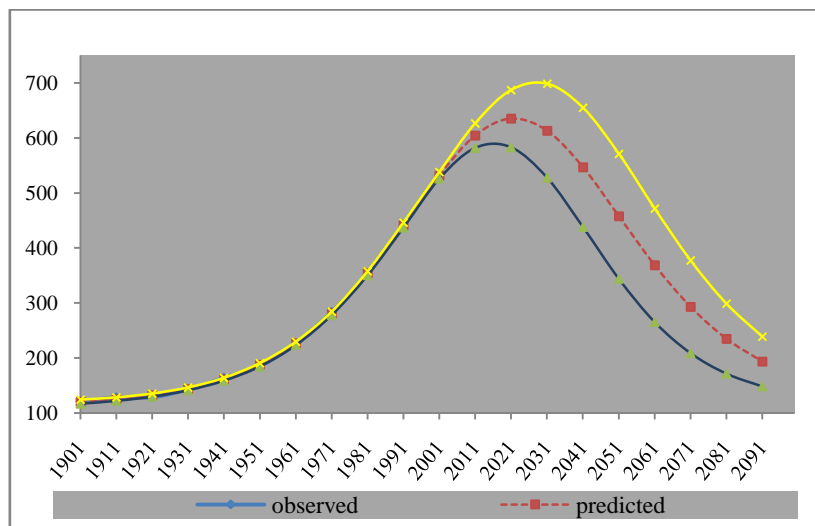


Fig 3: Logistic curve of population projection for Male

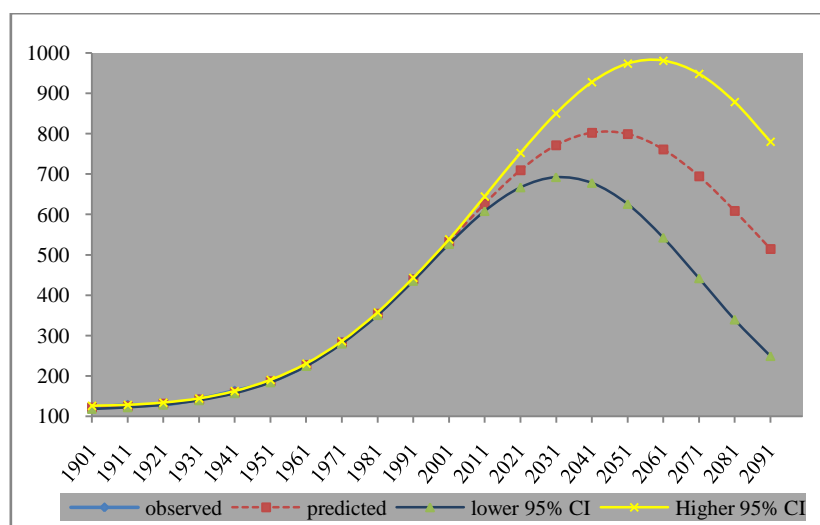


Fig 4: Gaussian curve of population projection for Male

The above Fig 3, logistic curve overlaps the observed data curve up to 2001 and preceding, the curve depicts that male population will stabilize earlier according to logistic curve than what it is according to Gaussian curve shown in Fig 4. The logistic curve drops rapidly but Gaussian curve

drops slowly after reaching saturation point. If we compare the three variants of both curves it is observed that the upper limit of Gaussian curve almost touches the level of 1000 but logistic curve upper limit touches up to 700 level.

Table 4: Projection for female population of India by Logistic and Gaussian curve

Year (x)	Population (y)	Logistic curve			Gaussian curve		
		Predicted population	95% Lower CI	95% Upper CI	Predicted population	95% Lower CI	95% Upper CI
1901	117.5	115.90	111.48	120.31	117.74	114.18	121.31
1911	123.7	120.30	116.57	124.03	120.81	117.84	123.78
1921	122.7	127.12	124.03	130.21	126.43	124.01	128.86
1931	136.0	137.61	134.81	140.42	136.19	133.89	138.49
1941	154.8	153.59	150.51	156.68	152.20	149.57	154.84
1951	175.5	177.56	173.97	181.16	177.06	174.13	179.98
1961	213.0	212.66	208.88	216.43	213.45	210.60	216.31
1971	264.1	262.18	258.51	265.86	263.65	260.79	266.51
1981	330.0	328.22	323.81	332.64	328.66	325.19	332.14
1991	405.8	409.00	403.90	414.09	407.39	403.79	411.00
2001	496.5	495.45	488.66	502.24	496.01	490.80	501.21
2011		569.82	543.70	595.95	587.75	570.99	604.51
2021		610.43	547.13	673.74	673.59	633.74	713.43
2031		603.02	494.91	711.13	743.50	668.35	818.65
2041		550.31	406.90	693.73	788.41	667.20	909.63
2051		470.09	313.44	626.73	802.11	628.50	975.71
2061		383.84	236.41	531.27	782.62	557.26	1007.98
2071		306.89	182.72	431.05	732.73	464.22	1001.24
2081		245.80	149.43	342.17	659.29	363.22	955.36
2091		200.87	130.45	271.29	571.69	267.84	875.53

* The values relate to populations in millions.

Same as above, the female population increases gradually from 115.9 by 1950 to 177.56 by 1951 but after that it increase rapidly from 212.66 by 1971 to 610.43 by 2021 and then a rapid fall can be observed i.e. from 610.43 by 2021 to 200.87 by 2091 according to logistic curve. Whereas, according to Gaussian curve it starts gradually same as logistic curve but attainment of peak of the curve is observed 802.11 by 2050 and at the end of the century it will be 571.69. The total female population will peak at 610.43 by 2021 and gets stability according to logistic curve whereas according to Gaussian its peak is attained at 802.11 by 2051 and gets stability. Female population will rapidly shrink at the end of the century by Logistic projection but by Gaussian curve it will shrink slowly. Opposing to the bitter condition of sex-ratio in India today it implies to a positive sign that sex ratio scenario in future will be in better condition than what is today because female population will more than male in future. Increasing sex-ratio, eliminating son-preference etc. endorse the aforesaid notion about female population. Total fertility rate of India has yet to reach fertility level so the projected stability for female population by logistic curve cannot be reached that earlier (i.e. nearby 2021). But in this context Gaussian curve project a reasonable forecast i.e. nearby 2041. One interesting fact come out of the picture that if compare the male and female population projections, we have more female population than male population. This may happen because of longer life expectancy of female than male. In Fig 5 for female population, each point meets up to 2001 but after that logistic curve drops with less breadth compared to Gaussian curve. Further, in Fig 5 stabilization takes place nearby 2031 but in Fig 6 (i.e. by Gaussian curve) it is attained nearby 2051. As we compare female projections stability of female population in both projections occurs after male population. And at the end of century female population will more than the population of males.

Conclusion

Population sizes and growth of a country directly affect the situation of policy, culture, economy and education as well, exploring the cost of natural sources. Two mathematical techniques

for predicting population growth have been examined to predict the population growth. The Gaussian model provides estimates better than Logistics model for Indian population. Findings of this study can be useful to the government and decision makers to create economic policies accordingly. Also these models have some limitations as there are differences in the actual population. These differences might be due to many reasons.

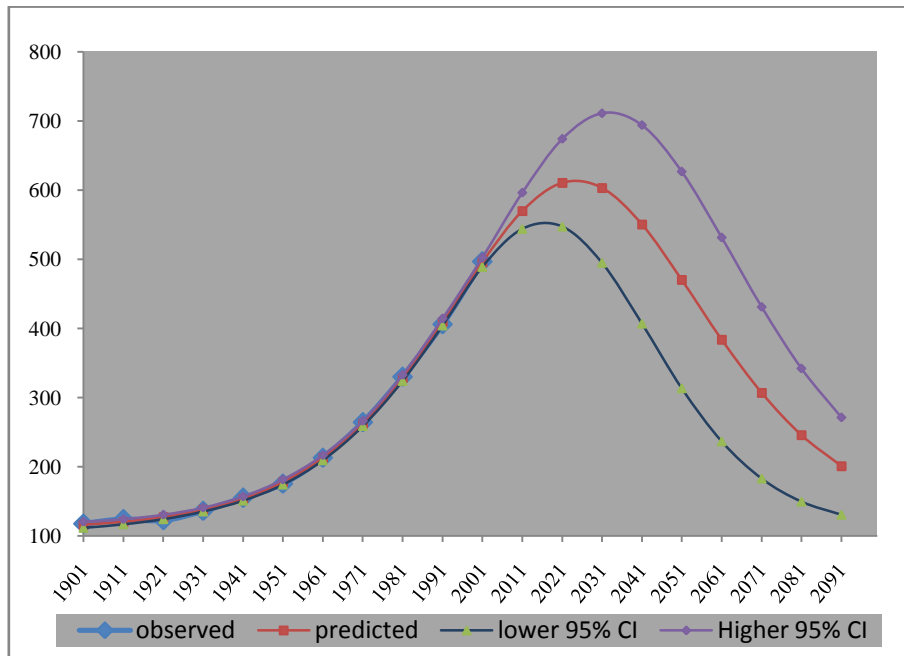


Fig 5: Logistic curve of population projection for female

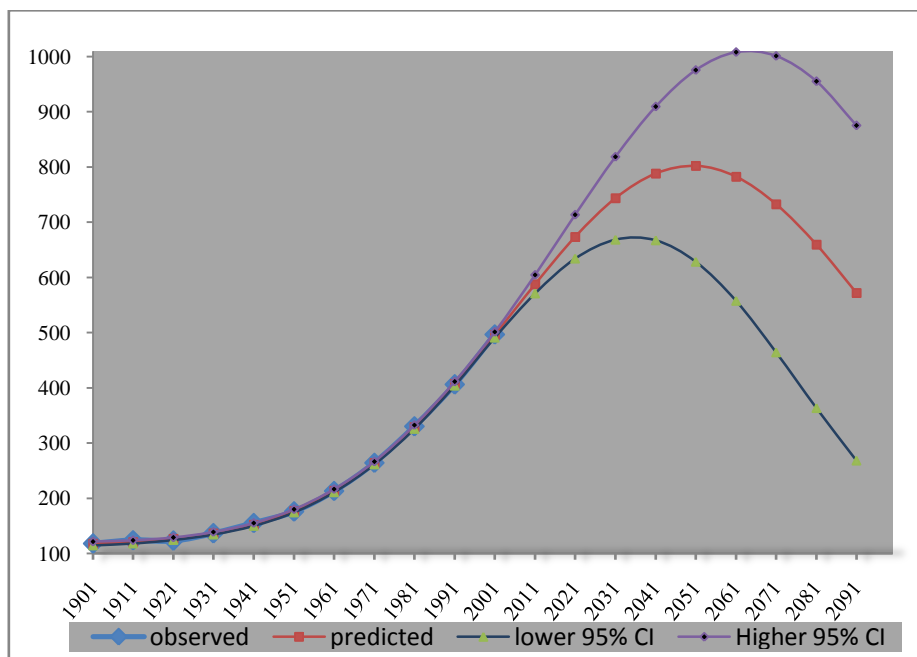


Fig 6: Gaussian curve of population projection for female

All the projections made by government, researchers and agencies for Indian population concludes that in recent future India will be the most populous country of world. Therefore, it is vital to figure out an effective model to predict the present and future population growth in India precisely, as well as its structure by applying the available data so as to adjust the speed and the component of population growth to the present all-round situation and meet the India's immense demand for continuous economic excellence.

Logistic growth curve effectiveness in real populations has limitations because the dynamics of populations is a complex phenomenon. In the classical paradigm Logistic model projection underestimates and show decline in total population very rapidly. Utilizing Gaussian model to population projection provides a more realistic summary of population projection. This study leads us to a make a notion about the applicability and appropriateness of Gaussian curve over logistic as well as other complex methods for fitting the population data and projecting the population. The proposed model yields over the Bayesian setup because it is less complex and projection is almost same so it is better to proceed with a less complex model with the same results. Smith and Sincich (1992) has already pointed out no evidence that complex and/or sophisticated techniques produce more accurate or less biased forecasts than simple, naive techniques.

The proposed Gaussian model estimated projection is shown in tables and a comprehensive comparison between several projections is made. Comprehensive comparison reveals that Gaussian model projection is better than Logistic model and much simpler to various methods used for population projections i.e. Bayesian method, cohort component method etc. And it is almost same with a few degree of variation to the projections made by several authors using different methods.

UN projections propose one view about future of India's population that medium scenario assumes that a TFR of 2.1 children would be reached by 2020–2025 and then level off at 1.85 by 2030–2035, resulting in a 2050 population of 1.6 billion (Carl Haub et al. 2006). The population of India is expected to be around 1.4 billion in 2025 (PNM Bhat: Demographic scenario, 2025) same inference can be drawn by our projection (by Gaussian curve) as it is 1.37 billion in 2021.

Our projection shows that Indian population will increase rapidly from 1.02 billion in 2001 to 1.57 billion by 2051, more half a billion population added in span of 50 years, and will attained peak at above 1.57 billion around 2051 (Figure 2). Total population will then start decline below rapidly 1.04 billion by the end of the century. Again, from the male and female population projection it is quite clear that female population will more than male population in future may be because life expectancy of female is more than male. If it will not more than male population certainly there will be approximately to 1:1 ratio. In other words, we can say, after mid of the 21st century, the population of India will enter an era of decline, ending the continuous increase in population. Based on the reference projection up to 2051 and thereafter, the long-term downward trend will continue. And by the end of the century country's population will plummet to that of population what is in the beginning of the century. Further, Gaussian curve can also be tested for several countries population projection as well as sub-national projections. It will be more suitable with fewer assumptions and less mathematical procedures.

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