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Research Article

On the Number of Menstrual Cycles Required for First Conception: An Insight of Chance Mechanism

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Abstract

The interval between marriage and first conception leading to a live birth plays an important role in determination of fertility of a female. One of the proximate determinants of natural fertility is fecundability, which is defined as the probability of conception that a married female will conceive during a month of exposure under unprotected cohabitations. Thus, number of menstrual cycles required for first conception may also be used to estimate the fecundability. In this study, under the certain assumptions a mixture of size biased geometric distribution and its modification have been proposed by considering two type of females having low and high risk of conception to estimate the fecundability. Comparison of proposed models with other conception time discrete models is attempted using some real data sets. To estimate parameters involved in the above models we have discussed both the methods, i.e., the method of moments and the maximum likelihood method.

Introduction

The interval between marriage and first conception leading to a live birth plays an important role in the determination of fertility component. This interval signifies couple's fertility at an early stage of married life. It is largely governed by fecundability of female because usually no female likes to use contraception to postpone the first birth in traditional Indian society. First birth interval is also free from the period of post-partum amenorrhea (PPA) associated with a live birth, whereas, other birth intervals are heavily affected by the fluctuations in PPA.

The representation of data on first birth interval to determine fecundability considering probability model has created the attention of demographers for over a period of time. Gini (1924) first used geometric distribution to estimate the mean fecundability from the data on number of menstrual cycle for first conception to a cohort of married females. Henry (1953) extended Gini's results and obtained the mean fecundability of the population and coefficient of variation from data on proportions of females conceiving during the first and the second months of exposure to the risk of conception.

It is well known fact that all females may not be biologically same in respect of reproductive performance. Keeping in view this fact, the variability in fecundability among females (Singh, 1964) derived a probability distribution for the waiting time for the first conception by treating time to be continuous and assuming one to one correspondence between a conception and a live birth. Potter and Parker (1964) & Sheps (1964) used Type-I geometric and compound geometric distribution respectively to describe data in the time of first conception, whereas, Singh (1961) proposed geometric and Type-I geometric for the same.

Singh (1982) have proposed a modified probability distribution for the waiting time to first conception taking into account premarital conceptions as well as the termination of study after a certain period of time. For the time of first birth Bhattacharya (1986) derived a model under the assumption that the exposure to the risk of conception is delayed due to visit of the females to her parent's house and hence the fecundability is less in the beginning and as age advances it reaches

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maximum and then decreases with increase in age. Later, Bhattacharya(1988) have proposed another time dependent model, which is more suitable for the societies where age at marriage is low.

To study the fertility of females several authors (Nagalinda, 1998; Booth, 2001) have investigated first birth interval for the females married with low age as well high age at marriage. The study of first birth interval, ascertained according to different age at marriage, is of greater importance for researchers. Also, this interval has been found to be significantly influenced by several socio-demographic variables (Singh, 1992; Nath, 1999). In this paper, the estimation of risk of first conception during one menstrual cycle using certain models for conception taking time to be discrete have been described in detail. Certain modifications of the model as well as comparison between the earlier model and the models which are derived with modification has been made and were described using real data sets. Throughout this paper it has been assumed that the fecundability of each female remains constant from month to month until pregnancy, and conception is a random event conditional on her fecundability. For simplicity throughout this paper one menstrual cycle equals to one month has been considered. The various models presented in this paper for comparison vary in their assumption regarding the distribution of fecundability among females.

Construction of Model

1. Model with Homogeneous Fecundability (Model-I)

Consider a homogeneous population in which all females have identical constant fecundability p. Let x, the month of conception is considered to be a random variable following a geometric distribution; i.e.

$$P[X = x] = pq^{x-1}; x = 1,2,\ldots \tag{1}$$

where, q=1-p. The probability that conception does not occur in the first x month is given as

$$P[X > x] = q^{x} \tag{2}$$

Thus, the conditional probability that a female conceives in month x given that she has not conceived in the first (x-1) months is given as

$$P[X = x \mid X > (x-1)] = p \tag{3}$$

The expression (3) shows that the conditional conception rate at the beginning of a month those who have not conceived till that time which is constant and equal to the probability p. In this case moment as well as Maximum Likelihood (ML) estimate will be same so ML estimate is represented in the table.

Estimation Procedure

Let x_1, x_2, ...,x_n denote a random sample of size n from the population. Let n_i, i=1,2,... ($\sum n_i = n$) be the observed frequency of conceptions in the i^{th} month and P_i, i = 1, 2,... be the probability of a conception in the i^{th} month. The likelihood function of the distribution given by expression (1) is given as

$$L(p) \propto \prod_{i=1} p(x_i)^{n_i} \tag{4}$$

In the present case the expression for the estimate of p will be same by both the methods, i.e., method of moments as well as method of maximum likelihood which is given as

$$\hat{p} = \frac{1}{\bar{x}} \tag{5}$$

where, $\bar{x}$ is the sample mean. It can also be shown that for large samples.

$$Var(\hat{p}) \approx \frac{p^2(1-p)}{N}, \tag{6}$$

Where N is the sample size. The data considered in this study clearly shows that most of the females conceives in their first menstrual cycle and, thus, the above model can be modified by taking inflated geometric distribution with homogeneous fecundability.

2. Inflated Model with Homogeneous Fecundability (Model-II)

In this model, it has been considered that in the population most of females conceive in their first menstrual cycle, i.e., in the first month, after marriage. In such a situation an inflated geometric distribution can be considered.

Let us assume that the proportion of females who did not conceive in first menstrual cycle after marriage be α and $(1-\alpha)$ is the remaining proportion of females who conceive in the first menstrual cycle after marriage. Keeping in view the above situation the probability density function of inflated geometric model is discussed as follows:

Consider a homogeneous population in which all females have identical constant fecundability p. Let x be the month of conception which is considered to be a random variable following a inflated geometric distribution; i.e.

$$P[X=x] = \begin{cases} 1-\alpha+\alpha p & \text{if} x = 1, \\ \alpha p q^{x-1} & \text{if} x = 2,3,\ldots. \end{cases} \tag{7}$$

where q=1-p. The probability that conception does not occur in the first x month is given by the following equation

$$P[X > x] = \alpha q^x \tag{8}$$

Thus, the conditional probability that a female conceives in month x given that she has not conceived in the first (x-1) month is given as

$$P[X = x \mid X > (x-1)] = p + \left(\frac{1}{\alpha(1-p)^{x-1}}\right) - (1-p)^{2-x} \tag{9}$$

The above expression shows that the conditional conception rate at the beginning of a month for those who have not conceived till that time is decreasing due to heterogeneity among females. In this case moment as well as ML estimate have been obtained and represented in the Table.

Moment Estimation Procedure

By equating the sample raw moments of the random variable X to the corresponding population moments, the moment estimators of α & p can be obtained as

$$\hat{\alpha} = \frac{p(\mu_1'-1)}{(1-p)} \quad \text{and} \quad \hat{p} = \frac{2(\mu_1'-1)}{(\mu_2'-\mu_1')}$$

Maximum Likelihood Estimation Procedure

Let x_1, x_2, ...,x_n denote a random sample of size n from the population, also n_1 denote the number of observations in first cell, n_2 the number of observations in second cell and n is the total number of observations. Thus, the likelihood function of the inflated geometric distribution is given by

$$L(P) \propto \prod_{i=1} P_i^{n_i} \tag{10}$$

$$L(P) = (1-\alpha+\alpha p)^{n_1}(\alpha p)^{n-n_1}(1-p)^{n\bar{X}-n} \tag{11}$$

Taking Log of equation (11) we have:

$$LogL = n_1 Log(1-\alpha+\alpha P)+(n-n_1)Log(\alpha p)+(n\overline{X}-n)Log(1-p) \qquad (12)$$

Now differentiating equation (12) with respect to α & p we obtain the following equations:

$$\frac{\partial LogL}{\partial \alpha} = \frac{n-n_1}{\alpha} - \frac{n_1(1-p)}{1-\alpha+\alpha p} \qquad (13)$$

$$\frac{\partial LogL}{\partial p} = \frac{n-n_1}{p} - \frac{n(\overline{X}-1)}{1-p} + \frac{n_1\alpha}{1-\alpha+\alpha p} \qquad (14)$$

By solving the above equations the estimate of α & p can be obtained and is given by the equations:

$$\hat{\alpha} = \frac{n-n_1}{n(1-p)} \quad \text{and} \quad \hat{p} = \frac{n-n_1}{n(\overline{X}-1)}$$

where $\overline{X}$ is the sample mean. Differentiating the equations (13) and (14) with respect to α & p we have:

$$\frac{\partial^2 LogL}{\partial \alpha^2} = -\frac{n_1(p-1)^2}{(1-\alpha+\alpha p)^2} - \frac{n-n_1}{\alpha^2} \qquad (15)$$

$$\frac{\partial^2 LogL}{\partial p^2} = -\frac{n_1\alpha^2}{(1-\alpha+\alpha p)^2} - \frac{n-n_1}{p^2} - \frac{n(\overline{X}-1)}{(1-p)^2} \qquad (16)$$

$$\frac{\partial^2 LogL}{\partial p \partial \alpha} = \frac{n_1}{(1-\alpha+\alpha p)^2} \qquad (17)$$

Now using the fact that

$$E(n_1) = n(1-\alpha+\alpha p) \qquad (18)$$

we obtain the element of the information matrix as-

$$\Phi_{11} = \frac{1}{n}E[\frac{-\partial^2 LogL}{\partial \alpha^2}] = \frac{1-p}{\alpha(1-\alpha+\alpha p)} \qquad (19)$$

$$\Phi_{22} = \frac{1}{n}E[\frac{-\partial^2 LogL}{\partial p^2}] = \frac{\alpha^2}{1-\alpha+\alpha p} + \frac{\alpha(1-p)}{p^2} + \frac{\overline{X}-1}{(1-p)^2} \qquad (20)$$

$$\Phi_{21} = \Phi_{12} = \frac{1}{n}E[\frac{-\partial^2 LogL}{\partial p \partial \alpha}] = \frac{1}{n}E[\frac{-\partial^2 LogL}{\partial \alpha \partial p}] = -\frac{1}{1-\alpha+\alpha p} \qquad (21)$$

Information Matrix can be shown as: I= $\begin{bmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{22} \end{bmatrix}$

and the variances and co-variance of the parameters are given as:

$$V(\alpha) = \frac{-\Phi_{22}}{\Phi_{11}\Phi_{22}-\Phi_{12}^2} \qquad (22)$$

$$V(p) = \frac{-\Phi_{11}}{\Phi_{11}\Phi_{22}-\Phi_{12}^2} \qquad (23)$$

$$Cov(\alpha, p) = \frac{\Phi_{12}}{\Phi_{11}\Phi_{22} - \Phi_{12}^2} \tag{24}$$

In this model the fecundability has been considered constant. It can be further modified by taking into consideration that the fecundability is not constant for all female and it varies female to female.

3. Model with Heterogeneous Fecundability (Model-III)

The model described by the equation (1) is modified by assuming that fecundability *p* is not constant for all females and it varies female to female. Let us assume that *p* is distributed among females with a general distribution *h*(*p*). Let us assume that among couples, fecundability is distributed as,

$$h(p) = \frac{1}{B(a,b)} p^{a-1}(1-p)^{b-1} \quad ;0 < p < 1 \quad \& \quad a,b > 0 \tag{25}$$

From equation (1) we have: $P[X = x \mid p] = pq^{x-1}; \quad x = 1,2,\ldots$ where *x* is the month of conception. Then, the unconditional distribution of *x* is given by

$$P[X = x] = \begin{cases} \frac{a}{a+b}, & \text{if} x{=}1, \frac{ab(b+1)\ldots(b+x-2)}{(a+b)(a+b+1)\ldots(a+b+x-1)}, \quad \text{if } x{=}2,3,\ldots. \end{cases} \tag{26}$$

The probability that conception does not occur in the first *x* months is given by the following equation

$$P[X > x] = \frac{b(b+1)\ldots(b+x-1)}{(a+b)(a+b+1)\ldots(a+b+x-1)}, \quad x \geq 1 \tag{27}$$

The conditional probability of conceiving at month *x* given that she has not conceived for the first (*x*-1) month is given as

$$P[X = x \mid X > (x-1)] = \frac{a}{a+b+x-1}, \quad x \geq 1 \tag{28}$$

The equation given by (28) is a decreasing function of *x* this means that the conceptions are occurring at a decreasing rate. Here, the variance of *X* can be obtained by the following equation:

$$Var(X) = \frac{ab(a+b-1)}{(a-1)^2(a-2)}, \tag{29}$$

In this case moment as well as ML estimate have been obtained and represented in the Table.

Moment Estimation Procedure

Potter and Parker (1964) obtain the moment estimators of *a* and *b*. Majumdar and Sheps (1970) give the asymptotic variance-covariance matrix of the moment estimators. By equating the sample raw moments of *X* to the corresponding population moments, we have obtained the moment estimators of a and b as:

$\hat{a} = \dfrac{2(\mu_2' - \mu_1'^2)}{\mu_2' - 2\mu_1'^2 + \mu_1'}$ and $\hat{b} = (\hat{a}-1)(\mu_1' - 1)$, where μ_i' denote the i^{th} sample moment about zero. The estimates exist only when μ_2' exists, i.e., $a > 2$.

Maximum Likelihood Estimation Procedure

Let x_1, x_2, ...,x_n denote a random sample of size n from the population. Let P_i, i = 1, 2,... be the probability of a conception in i^{th} month. Let n_i, i=1,2,... ($\sum n_i = n$) be the observed frequency of conceptions in i^{th} month. The likelihood functions for the sample of *n* values is equal to

$$L(P) \propto \prod_{i=1} P_i^{n_i} \quad (30)$$

Taking log of equation (30) and differentiating partially with respect to *a* & *b* we have:

$$\frac{\partial LogL}{\partial a} = \sum_{i=1} n_i \frac{\partial LogP_i}{\partial a} \tag{31}$$

$$\frac{\partial LogL}{\partial b} = \sum_{i=1} n_i \frac{\partial LogP_i}{\partial b} \tag{32}$$

where

$$\frac{\partial LogP_x}{\partial a} = \frac{1}{a} - \sum_{j=0}^{x-1} \frac{1}{a+b+j}, \quad x \geq 1 \tag{33}$$

$$\frac{\partial LogP_x}{\partial b} = \begin{cases} \frac{-1}{a+b}, & \text{for} x=1, \sum_{j=0}^{x-2} \frac{1}{b+j} - \sum_{j=0}^{x-1} \frac{1}{a+b+j}, \quad \text{for } x=2,3,\ldots. \end{cases} \tag{34}$$

The maximum likelihood estimates of parameter *a*, *b* can be obtained by solving the simultaneous equations given by (31) and (32). The element of information matrix are:

$$\Phi_{11} = n \sum_{i=1} P_i \frac{\partial^2 LogP_i}{\partial a^2} \tag{35}$$

$$\Phi_{22} = n \sum_{i=1} P_i \frac{\partial^2 LogP_i}{\partial b^2} \tag{36}$$

$$\Phi_{21} = \Phi_{12} = n \sum_{i=1} P_i \frac{\partial^2 LogP_i}{\partial b \partial a} \tag{37}$$

where

$$\frac{\partial^2 LogP_x}{\partial a^2} = -\frac{1}{a^2} + \sum_{j=0}^{x-1} \frac{1}{(a+b+j)^2}, \quad x \geq 1 \tag{38}$$

$$\frac{\partial^2 LogP_x}{\partial b^2} = \begin{cases} \frac{1}{(a+b)^2}, & \text{for} x=1, -\sum_{j=0}^{x-2} \frac{1}{(b+j)^2} + \sum_{j=0}^{x-1} \frac{1}{(a+b+j)^2}, \quad \text{for } x=2,3,\ldots. \end{cases} \tag{39}$$

$$\frac{\partial^2 LogP_x}{\partial a \partial b} = \sum_{j=0}^{x-1} \frac{1}{(a+b+j)^2}, \quad x \geq 1 \tag{40}$$

Information Matrix can be shown as: I= $\begin{bmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{22} \end{bmatrix}$

Modification of this model can be introduced by taking into consideration a mixture model which describes that there are two type of females in population with respect to fecundability. One group of female having low fecundability and another group of females having high fecundability.

4. Mixture Model (Model-IV)

Let the couple have *k* menstrual cycles, i.e., equal to *k* month until the first pregnancy occurs. Let us assume *k* to be small so that the aging during the follow-up interval will have negligible effects on the fecundability. Weinberg & Gladden (1986) have given a probability model assuming fecundability of conception to be constant throughout the first birth interval but these probabilities vary across couples and follow beta distribution. In this study, a mixture of two geometric distribution has been considered under the assumption that there are two types of females present in the population having low and high risk of conception say p and θ with proportion α and $1-\alpha$ respectively in the population. Since p and θ are the probability of conception and lie between 0 and 1 and it is assumed that they remain constant within the group during the exposure period.

Under the assumption, let x be a random variable which is the number of months required for first conception for each females in the population, and it follows mixture of two geometric distributions whose expression is given as

$$P[X=x]=\alpha p(1-p)^{x-1}+(1-\alpha)\theta(1-\theta)^{x-1}; \quad x=1,2,\ldots \tag{41}$$

The probability that conception does not occur in the first x month is given by the following equation

$$P[X>x]=\alpha(1-p)^{x}+(1-\alpha)(1-\theta)^{x} \tag{42}$$

The conditional probability that a female conceives in month x given that she has not conceived in the first (x-1) months is given by

$$P[X=x \mid X>(x-1)]=\frac{\alpha p(1-p)^{x-1}+(1-\alpha)\theta(1-\theta)^{x-1}}{\alpha(1-p)^{x-1}+(1-\alpha)(1-\theta)^{x-1}} \tag{43}$$

The expression (43) shows that the conditional conception rate at the beginning of a month those who have not conceived till that time. In this case moment as well as ML estimate have been obtained and represented in the Table.

Moment Estimation Procedure

The parameters involved in the model given by equation (41) p, θ and α have been estimated by method of moment (MM). For the purpose of estimation of the parameters through MM method, the first three raw moments are given as:

$$E(x)=\mu_1'=\frac{\alpha}{p}+\frac{(1-\alpha)}{\theta} \tag{44}$$

$$E(x^2)=\mu_2'=\alpha[\frac{2-p}{p^2}]+(1-\alpha)[\frac{2-\theta}{\theta^2}] \tag{45}$$

$$E(x^3)=\mu_3'=\alpha[\frac{p^2+6-6p}{p^3}]+(1-\alpha)[\frac{\theta^2+6-6\theta}{\theta^3}] \tag{46}$$

Using the equations (44), (45) and (46) we can have expression of α and p as:

$$\hat{\alpha}=p[\frac{\mu_1'\theta-1}{\theta-p}] \tag{47}$$

$$\hat{p}=2[\frac{\mu_1'\theta-1}{\mu_2'\theta+\mu_1'\theta-2\mu_1'}] \tag{48}$$

$$\theta^2(2\mu_1'\mu_3'+\mu_1'^2-3\mu_2'^2)+\theta(6\mu_1'\mu_2'+6\mu_1'^2-2\mu_3'-6\mu_2'-4\mu_1')-12\mu_1'^2+6\mu_2'+6\mu_1'=0 \tag{49}$$

The above cubic equation can be easily solved in order to get an estimate of θ. After getting value of θ, we can easily obtain the estimate of p and α by using equation (48) and (47) respectively. Because of the high sampling variance of the third moment, one may look for some other function to replace it. The other suitable method is known as Zero frequency method (Karlis and Xekalaki, 2003). When the frequency count in first cell is relatively high a usual choice for discrete distributions is the zero relative frequency of the data set which is equated to the probability of zero under the assumed distribution (Kemp and Kemp, 1988).

Replacing the third moment equation by the zero frequency equation, the estimation can be improved, mainly in efficiency, when the sample mean is small. This fact makes the method of zero frequency an interesting alternative to the Maximum likelihood having for small values of the mean. So, the resulting system of equations is:

$$E(x) = \mu_1^{'} = \frac{\alpha}{p} + \frac{(1-\alpha)}{\theta} \quad (50)$$

$$E(x^2) = \mu_2^{'} = \alpha[\frac{2-p}{p^2}] + (1-\alpha)[\frac{(2-\theta)}{\theta^2}] \quad (51)$$

$$\frac{n_0}{n} = \alpha p + (1-\alpha)\theta \quad (52)$$

where n_0 is the observed proportion of value in the first cell in the sample. For distributions with a high probability in the first cell, this method is expected to work satisfactorily. Moreover, due to the lower variance of the proportion in the first cell relative to that of the third sample moment, the method is expected to have a higher efficiency. Using equations (50), (51) and (52) we have:

$$\hat{\alpha} = p[\frac{\mu_1^{'}\theta - 1}{\theta - p}] \quad (53)$$

$$\hat{p} = 2[\frac{\mu_1^{'}\theta - 1}{\mu_2^{'}\theta + \mu_1^{'}\theta - 2\mu_1^{'}}] \quad (54)$$

$$\theta^2(\mu_2^{'}n + \mu_1^{'}n - 2\mu_1^{'2}n) + \theta(2\mu_1^{'}n - \mu_2^{'}n_0 - \mu_1^{'}n_0) - 2n_0\mu_1^{'} - 2n = 0 \quad (55)$$

The above cubic equation can be easily solved in order to get an estimate of θ. After getting value of θ, we can easily obtain the estimate of p and α by using equation (54) and (53) respectively.

Maximum Likelihood Estimation Procedure

The likelihood function of the model 41 can be written as

$$L(p) \propto \prod_{i=1} p(x_i)^{n_i} \quad (56)$$

$$L(p) = [\alpha p q^{x_1-1} + (1-\alpha)\theta q'^{x_1-1}]^{n_1}[\alpha p q^{x_2-1} + (1-\alpha)\theta q'^{x_2-1}]^{n_2}[q'^2 - \alpha(p-\theta)(2-p-\theta)]^{n-n_1-n_2} \quad (57)$$

$$LogL = n_1 log[\alpha p + (1-\alpha)\theta] + n_2 log[\alpha pq + (1-\alpha)\theta q'] + (n-n_1-n_2) log[q'^2 - \alpha(p-\theta)(2-p-\theta)] \quad (58)$$

Now differentiating the above equations with respect to α , p and θ following equations are obtained

$$\frac{\partial LogL}{\partial\alpha} = \frac{n_1(p-\theta)}{\alpha p + (1-\alpha)\theta} + \frac{n_2(pq-\theta q')}{\alpha pq + (1-\alpha)\theta q'} - \frac{(n-n_1-n_2)(p-\theta)(2-p-\theta)}{q'^2 - \alpha(p-\theta)(2-p-\theta)} \quad (59)$$

$$\frac{\partial^2 LogL}{\partial\alpha^2} = -\frac{n_1(p-\theta)^2}{(\alpha p + (1-\alpha)\theta)^2} - \frac{n_2(pq-\theta q')^2}{(\alpha pq + (1-\alpha)\theta q')^2} - \frac{(n-n_1-n_2)(p-\theta)^2(2-p-\theta)^2}{(q'^2 - \alpha(p-\theta)(2-p-\theta))^2}$$

(60)

$$\frac{\partial LogL}{\partial p}=\frac{n_1\alpha}{\alpha p+(1-\alpha)\theta}+\frac{n_2\alpha(1-2p)}{\alpha pq+(1-\alpha)\theta q'}-\frac{(n-n_1-n_2)2\alpha q}{q'^2-\alpha(p-\theta)(2-p-\theta)} \quad (61)$$

$$\frac{\partial^2 LogL}{\partial p^2}=-\frac{n_1\alpha^2}{(\alpha p+(1-\alpha)\theta)^2}-\frac{n_2\alpha^2(1-2p)^2}{(\alpha pq+(1-\alpha)\theta q')^2}-\frac{(n-n_1-n_2)4\alpha^2 q^2}{(q'^2-\alpha(p-\theta)(2-p-\theta))^2} \quad (62)$$

$$\frac{\partial LogL}{\partial \theta}=\frac{n_1(1-\alpha)}{\alpha p+(1-\alpha)\theta}+\frac{n_2(1-\alpha)(1-2\theta)}{\alpha pq+(1-\alpha)\theta q'}+\frac{(n-n_1-n_2)(2\alpha+2\theta-2-2\alpha\theta)}{q'^2-\alpha(p-\theta)(2-p-\theta)} \quad (63)$$

$$\frac{\partial^2 LogL}{\partial \theta^2}=-\frac{n_1(1-\alpha)^2}{(\alpha p+(1-\alpha)\theta)^2}-\frac{n_2(1-\alpha)^2(1-2\theta)^2}{(\alpha pq+(1-\alpha)\theta q')^2}-\frac{(n-n_1-n_2)(2\alpha+2\theta-2-2\alpha\theta)^2}{(q'^2-\alpha(p-\theta)(2-p-\theta))^2}$$

(64)

Solving equations (59), (61) and (63) under the following conditions we get the estimate of $\hat{\alpha}$, $\hat{p}$ and $\hat{\theta}$.

$p<\theta,\ \theta>\frac{1}{2},\ p<\frac{1}{2}$ and $p+\theta<1$

Now using the fact that

$$E(n_i)=nP_i \quad (65)$$

$$E(n_1)=n[\alpha p+(1-\alpha)\theta] \quad (66)$$

$$E(n_2)=n[\alpha pq+(1-\alpha)\theta q'] \quad (67)$$

$$E(n-n_1-n_2)=n[q'^2-\alpha(p-\theta)(2-p-\theta)] \quad (68)$$

The element of information matrix are:

$$\Phi_{11}=E[-\frac{\partial^2 LogL}{\partial \alpha^2}] \quad (69)$$

$$\Phi_{22}=E[-\frac{\partial^2 LogL}{\partial p^2}] \quad (70)$$

$$\Phi_{33}=E[-\frac{\partial^2 LogL}{\partial \theta^2}] \quad (71)$$

$$\Phi_{12}=\Phi_{21}=E[-\frac{\partial^2 LogL}{\partial \alpha\partial p}]=E[-\frac{\partial^2 LogL}{\partial p\partial \alpha}] \quad (72)$$

$$\Phi_{13}=\Phi_{31}=E[-\frac{\partial^2 LogL}{\partial \alpha\partial \theta}]=E[-\frac{\partial^2 LogL}{\partial \theta\partial \alpha}] \quad (73)$$

$$\Phi_{23}=\Phi_{32}=E[-\frac{\partial^2 LogL}{\partial p\partial \theta}]=E[-\frac{\partial^2 LogL}{\partial \theta\partial p}] \quad (74)$$

where,

$$\frac{\partial^2 LogL}{\partial \alpha\partial p}=\frac{n_1\theta}{(\alpha p+(1-\alpha)\theta)^2}+\frac{n_2\theta q'(1-2p)}{(\alpha pq+(1-\alpha)\theta q')^2}-\frac{(n-n_1-n_2)(2qq'^2)}{(q'^2-\alpha(p-\theta)(2-p-\theta))^2} \quad (75)$$

$$\frac{\partial^2 LogL}{\partial\alpha\partial\theta} = -\frac{n_1 p}{(\alpha p+(1-\alpha)\theta)^2} - \frac{n_2 pq(1-2\theta)}{(\alpha pq+(1-\alpha)\theta q')^2} + \frac{2q^2 q'(n-n_1-n_2)}{(q'^2-\alpha(p-\theta)(2-p-\theta))^2} \quad (76)$$

$$\frac{\partial^2 LogL}{\partial p\partial\theta} = -\frac{n_1\alpha(1-\alpha)}{(\alpha p+(1-\alpha)\theta)^2} - \frac{n_2\alpha(1-\alpha)(1-2p)(1-2\theta)}{(\alpha pq+(1-\alpha)\theta q')^2} + \frac{4\alpha qq'(n-n_1-n_2)(1-\alpha)}{(q'^2-\alpha(p-\theta)(2-p-\theta))^2}$$

(77)

where,

$$\frac{\partial^2 LogL}{\partial p\partial\alpha} = \frac{\partial^2 LogL}{\partial\alpha\partial p}, \quad \frac{\partial^2 LogL}{\partial\theta\partial\alpha} = \frac{\partial^2 LogL}{\partial\alpha\partial\theta} \text{ and } \frac{\partial^2 LogL}{\partial\theta\partial p} = \frac{\partial^2 LogL}{\partial p\partial\theta}$$

Information Matrix can be shown as: $I=\begin{bmatrix}\Phi_{11} & \Phi_{12} & \Phi_{13}\\ \Phi_{21} & \Phi_{22} & \Phi_{23}\\ \Phi_{31} & \Phi_{32} & \Phi_{33}\end{bmatrix}$

Modification of this model can be done by introducing the heterogeneity among females with respect to their fecundabilty.

5. Mixture heterogeneous Model (Model-V)

In previous model it has been assumed that there are two types of females present in the population having low and high risk of conception say p and θ with proportion α and $1-\alpha$ respectively in the population. The extension of the above model has been done by assuming that among couples, fecundability is distributed as $h(p)$ and $g(\theta)$ with their risk of conception as p and θ respectively which are given as

$$h(p) = \frac{1}{B(a_1,b_1)} p^{a_1-1}(1-p)^{b_1-1} \quad ;0<p<1 \quad \& \quad a_1,b_1>0 \quad (78)$$

$$g(\theta) = \frac{1}{B(a_2,b_2)} \theta^{a_2-1}(1-\theta)^{b_2-1} \quad ;0<\theta<1 \quad \& \quad a_2,b_2>0 \quad (79)$$

Now, $P[X=x\,|\,p,\theta] = \alpha p(1-p)^{x-1} + (1-\alpha)\theta(1-\theta)^{x-1}; \quad x=1,2,\ldots$

where x is the month of conception and

$$P[X,p,\theta] = \frac{[\alpha p(1-p)^{x-1}+(1-\alpha)\theta(1-\theta)^{x-1}]p^{a_1-1}(1-p)^{b_1-1}\theta^{a_2-1}(1-\theta)^{b_2-1}}{B(a_1,b_1)B(a_2,b_2)} \quad (80)$$

Integrating equation (46) over p and θ respectively we have the unconditional distribution of the conception month as

$$P[X=x] = \begin{cases} \dfrac{\alpha B(a_1+1,b_1)}{B(a_1,b_1)} + \dfrac{(1-\alpha)B(a_2+1,b_2)}{B(a_2,b_2)}, \text{ if } x=1 \\[2ex] \dfrac{\alpha B(a_1+1,x+b_1-1)}{B(a_1,b_1)} + \dfrac{(1-\alpha)B(a_2+1,x+b_2-1)}{B(a_2,b_2)}, \text{ if } x=2,3,.... \end{cases}$$

(81)

The probability that conception does not occur in the first x month is given as

$$P[X>x] = \frac{\alpha b_1(b_1+1)\ldots(b_1+x-1)}{(a_1+b_1)\ldots(a_1+b_1+x-1)} + \frac{(1-\alpha)b_2(b_2+1)\ldots(b_2+x-1)}{(a_2+b_2)\ldots(a_2+b_2+x-1)} \quad (82)$$

The conditional probability of conceiving at month x given that she has not conceived for the first (x-1) months is given as

$$P[X > x \mid X > (x-1)] = \frac{[\frac{\alpha a_1 b_1 (b_1+1)\dots(b_1+x-2)}{(a_1+b_1)\dots(a_1+b_1+x-1)}]+[\frac{(1-\alpha)a_2 b_2(b_2+1)\dots(b_2+x-2)}{(a_2+b_2)\dots(a_2+b_2+x-1)}]}{[\frac{\alpha b_1(b_1+1)\dots(b_1+x-2)}{(a_1+b_1)\dots(a_1+b_1+x-2)}]+[\frac{(1-\alpha)b_2(b_2+1)\dots(b_2+x-2)}{(a_2+b_2)\dots(a_2+b_2+x-2)}]} \quad (83)$$

Maximum Likelihood Estimation Procedure

Moment estimators of this distribution are very difficult to obtain, since they involve a fifth degree equation. Thus, Maximum likelihood estimation procedure has been considered. let n_i, i=1,2,... be the observed frequency of conceptions in month i. The likelihood function for the sample of $\sum n_i = n$ values is proportional to:

$$L(P) \propto \prod_{i=1} P_i^{n_i} \quad (84)$$

and $LogL = \sum_{i=1} n_i Logp_i$ (85)

Now differentiating equation (85) with respect to a_1, b_1, a_2, b_2 and α respectively we have following equations:

$$\frac{\partial LogL}{\partial a_1} = \sum n_i \frac{\partial Logp_i}{\partial a_1} = \sum \frac{n_i}{p_i}\frac{\partial p_i}{\partial a_1} \quad (86)$$

$$\frac{\partial LogL}{\partial b_1} = \sum n_i \frac{\partial Logp_i}{\partial b_1} = \sum \frac{n_i}{p_i}\frac{\partial p_i}{\partial b_1} \quad (87)$$

$$\frac{\partial LogL}{\partial a_2} = \sum n_i \frac{\partial Logp_i}{\partial a_2} = \sum \frac{n_i}{p_i}\frac{\partial p_i}{\partial a_2} \quad (88)$$

$$\frac{\partial LogL}{\partial b_2} = \sum n_i \frac{\partial Logp_i}{\partial b_2} = \sum \frac{n_i}{p_i}\frac{\partial p_i}{\partial b_2} \quad (89)$$

$$\frac{\partial LogL}{\partial \alpha} = \sum n_i \frac{\partial Logp_i}{\partial \alpha} = \sum \frac{n_i}{p_i}\frac{\partial p_i}{\partial \alpha} \quad (90)$$

Let us assume $p_i = \alpha p_{i1} + (1-\alpha) p_{i2}$ (91)

where,

$$p_{ij} = \begin{cases} \frac{a_j}{a_j+b_j}, & \text{if i=1;j=1,2,} \\ \frac{a_j b_j (b_j+1)\dots(b_j+i-2)}{(a_j+b_j)\dots(a_j+b_j+i-1)}, & \text{if i=2,3,…;j=1,2.} \end{cases} \quad (92)$$

Again $\frac{\partial p_i}{\partial a_1} = \alpha \frac{\partial p_{i1}}{\partial a_1} = \alpha p_{i1} \frac{\partial Logp_{i1}}{\partial a_1}$ (93)

$$\frac{\partial p_i}{\partial b_1} = \alpha \frac{\partial p_{i1}}{\partial b_1} = \alpha p_{i1} \frac{\partial Logp_{i1}}{\partial b_1} \quad (94)$$

$$\frac{\partial p_i}{\partial a_2} = (1-\alpha)\frac{\partial p_{i2}}{\partial a_2} = (1-\alpha)\, p_{i2}\,\frac{\partial Logp_{i2}}{\partial a_2} \tag{95}$$

$$\frac{\partial p_i}{\partial b_2} = (1-\alpha)\frac{\partial p_{i2}}{\partial b_2} = (1-\alpha)\, p_{i2}\,\frac{\partial Logp_{i2}}{\partial b_2} \tag{96}$$

$$\frac{\partial p_i}{\partial \alpha} = p_{i1} - p_{i2} \tag{97}$$

These equations can be solved using following relations-

$$\frac{\partial p_{ij}}{\partial a_j} = \frac{1}{a_j} - \sum_{t=0}^{i-1}\frac{1}{a_j + b_j + t}, \quad i \ge 1; j = 1,2 \tag{98}$$

$$\frac{\partial p_{ij}}{\partial b_j} = \begin{cases} \dfrac{-1}{a_j + b_j}, \quad \text{ifi} = 1; \text{j} = 1,2, \sum_{t=0}^{i-2}\dfrac{1}{b_j + t} - \sum_{t=0}^{i-1}\dfrac{1}{a_j + b_j + t}, \quad \text{ifi} \ge 2; \text{j} = 1,2. \end{cases} \tag{99}$$

The equations for obtaining the maximum likelihood estimates are solved using some method of Numerical Analysis. For the purpose of solving we need to have a set of initial estimates. Moment estimators of this distribution are very difficult to obtain, since they involve a fifth degree equation. Estimators based on observed proportions as derived in the single beta distribution model are also hard to find. The following guidelines are used to obtain initial estimates. An initial guess is made by the means of the two mixing distributions. This can be done graphically by plotting the empirical cumulative distribution function. An approximate value of the mixing proportion also can be read from the graph. Using these guesses and two observed proportions all parameters can be approximated (Suchindran, 1972).

Application of the Models

For estimation, comparison and interpretation of parameters, the data has been taken from Baired (1985) where pregnant females were asked how many menstrual cycles it took them to get pregnant. Only those females who conceived within 21 menstrual cycles are considered for this study. Here the females are classified into two groups on the characterizing that whether they are smoker or not. A total of 586 females have been considered in this study. In this section the estimate of parameters involved in the above described models and their fitting have been discussed. Mainly two estimation procedures, i.e., Method of moments and Maximum likelihood have been used to estimate the parameters for comparison purpose. For the model described in the previous section the expected frequencies for all the models are obtained after estimating the parameters involved in the model. Table 1 to 3 show the expected frequencies along with the observed frequencies. The value of χ^2 with degree of freedom are also given in the respective tables. The value of χ^2 at 5 percent level of significance shown in the tables clearly indicate that Model-IV and Model-V fit the data quite well as compared to rest of the models. In Table 3 expected frequencies are not given for Model-IV since the model does not full-fill the condition $(p + \theta < 1)$ which is necessary here for the estimation of the parameters through Maximum likelihood method.

After analyzing the data, estimates of proposed model-IV are shown in Table 4, which clearly reveals that among the total female population considered in data, 59 percent females have risk of conception of 0.25, whereas, the rest 41 percent female have risk of conception 0.58. It is worthwhile to mention here that the average of geometric distribution (started from one) is reciprocal of the risk or probability. Thus, from the result, it can be concluded that 59 percent females take on average 4 menstrual cycles to conceive, whereas, 41 percent females take only on average 2 menstrual cycles to conceive. Results from the parameters involved in the homogeneous model show that females take on average 3 menstrual cycles to get first conception, whereas, from the parameters involved in the inflated model it is observed that 16 percent females conceive in their first menstrual cycle after marriage.

From Table 4, it is clear that 51 percent non-smoker females have risk of conception 0.26, whereas 49 percent female’s risk of conception is 0.56. Thus, for non-smoker females in the population, it can be said that 51 percent females take on average 4 menstrual cycles to conceive, whereas, 49 percent females take on average 2 menstrual cycles to get pregnant. According to homogeneous model it is found that females take on average 3 menstrual cycles to get first conception, whereas, according to inflated model it is observed that 16 percent females conceive in their first menstrual cycle after marriage. Similarly, Table 4 shows that among the smoker female population considered in data, 89 percent females have risk of conception of 0.21, whereas, only 10 percent female’s risk of conception is 0.92. Thus, it can be said that 89 percent females take on average 5 menstrual cycles to conceive, whereas, only 10 percent of females take on average 1 menstrual cycle to conceive. Results show that according to homogeneous model females take on average 5 menstrual cycles to get first conception, whereas, according to inflated model it is observed that 11 percent females conceive in their first menstrual cycle after marriage. After application of the models on the given data, estimates show that smoking delays the conception through increasing the infertility (Baired, 1985; Weinberg & Gladden, 1986). Further, it is observed that the model proposed by us which are model-IV and model-V give better fit than Model-I, Model-II and Model-III to get the estimate of risk of conception and consequently number of menstrual cycles required for first conception.

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Table 1: Observed and expected distribution of number of menstrual cycles required for the first conception (total females)

<table>
<tr><th rowspan="3">No. of menstrual cycle</th><th rowspan="3">Observed frequency</th><th colspan="5">Expected frequency</th><th colspan="3">Expected frequency</th></tr>
<tr><th colspan="5">(ML Method)</th><th colspan="3">MM Method</th></tr>
<tr><th>Model-I</th><th>Model-II</th><th>Model-III</th><th>Model-IV</th><th>Model-V</th><th>Model-II</th><th>Model-III</th><th>Model-IV</th></tr>
<tr><td>1</td><td>227</td><td>192.95</td><td>227.00</td><td>191.59</td><td>227.00</td><td>227.00</td><td>265.95</td><td>184.83</td><td>226.99</td></tr>
<tr><td>2</td><td>123</td><td>129.42</td><td>96.90</td><td>122.06</td><td>123.00</td><td>123.00</td><td>78.02</td><td>124.81</td><td>118.36</td></tr>
<tr><td>3</td><td>72</td><td>86.81</td><td>70.75</td><td>84.68</td><td>72.67</td><td>65.80</td><td>58.48</td><td>84.83</td><td>68.80</td></tr>
<tr><td>4</td><td>42</td><td>58.22</td><td>51.65</td><td>56.97</td><td>46.40</td><td>44.12</td><td>44.41</td><td>58.01</td><td>44.23</td></tr>
<tr><td>5</td><td>21</td><td>39.05</td><td>37.71</td><td>38.61</td><td>31.47</td><td>29.50</td><td>33.72</td><td>39.91</td><td>30.71</td></tr>
<tr><td>6</td><td>31</td><td>26.19</td><td>27.53</td><td>26.36</td><td>22.23</td><td>22.42</td><td>25.61</td><td>27.62</td><td>22.42</td></tr>
<tr><td>7</td><td>11</td><td>17.57</td><td>20.10</td><td>18.13</td><td>16.12</td><td>15.53</td><td>19.45</td><td>18.12</td><td>16.86</td></tr>
<tr><td>8</td><td>14</td><td>11.78</td><td>14.67</td><td>12.55</td><td>12.87</td><td>12.90</td><td>14.77</td><td>13.45</td><td>12.89</td></tr>
<tr><td>9</td><td>6</td><td>7.90</td><td>10.71</td><td>8.74</td><td>8.82</td><td>10.96</td><td>11.21</td><td>9.46</td><td>9.95</td></tr>
<tr><td>10</td><td>4</td><td rowspan="2">8.86</td><td rowspan="3">9.43</td><td>6.13</td><td rowspan="2">11.52</td><td>7.23</td><td>8.51</td><td>6.69</td><td rowspan="2">13.70</td></tr>
<tr><td>11</td><td>7</td><td>5.32</td><td>5.94</td><td>6.47</td><td>4.76</td></tr>
<tr><td>12</td><td>9</td><td rowspan="6">7.24</td><td rowspan="2">7.26</td><td rowspan="3">8.57</td><td rowspan="2">8.29</td><td rowspan="3">11.47</td><td rowspan="2">5.83</td><td rowspan="3">11.12</td></tr>
<tr><td>13</td><td>3</td><td rowspan="5">6.01</td></tr>
<tr><td>14</td><td>4</td><td rowspan="4">7.60</td><td rowspan="3">7.25</td><td rowspan="4">7.67</td></tr>
<tr><td>15</td><td>2</td><td rowspan="3">6.31</td><td rowspan="3">7.93</td><td rowspan="3">9.96</td></tr>
<tr><td>16</td><td>3</td></tr>
<tr><td>17+</td><td>7</td><td>6.07</td></tr>
<tr><td>Total</td><td>586</td><td>586</td><td>586</td><td>586</td><td>586</td><td>586</td><td>586</td><td>586</td><td>586</td></tr>
<tr><td colspan="2">χ^2</td><td>85.93</td><td>35.62</td><td>38.70</td><td>11.86</td><td>13.94</td><td>53.27</td><td>47.19</td><td>13.59</td></tr>
<tr><td colspan="2">d.f.</td><td>9</td><td>10</td><td>10</td><td>8</td><td>8</td><td>10</td><td>10</td><td>8</td></tr>
</table>

Table 2: Observed and expected distribution of number of menstrual cycles required for the first conception (non-smoker females)

<table>
<tr><th rowspan="3">No. of menstrual cycle</th><th rowspan="3">Observed frequency</th><th colspan="5">Expected frequency</th><th colspan="3">Expected frequency</th></tr>
<tr><th colspan="5">(ML Method)</th><th colspan="3">MM Method</th></tr>
<tr><th>Model-I*</th><th>Model-II</th><th>Model-III</th><th>Model-IV</th><th>Model-V**</th><th>Model-II</th><th>Model-III</th><th>Model-IV</th></tr>
<tr><td>1</td><td>198</td><td>160.02</td><td>198.00</td><td>183.48</td><td>198.00</td><td>198.00</td><td>234.76</td><td>165.65</td><td>198.00</td></tr>
<tr><td>2</td><td>107</td><td>107.33</td><td>83.78</td><td>105.91</td><td>107.00</td><td>107.46</td><td>63.76</td><td>107.34</td><td>102.45</td></tr>
<tr><td>3</td><td>55</td><td>71.99</td><td>59.41</td><td>68.27</td><td>61.55</td><td>55.73</td><td>47.58</td><td>70.18</td><td>57.72</td></tr>
<tr><td>4</td><td>38</td><td>48.29</td><td>42.13</td><td>42.74</td><td>37.66</td><td>34.23</td><td>35.50</td><td>46.29</td><td>35.53</td></tr>
<tr><td>5</td><td>18</td><td>32.39</td><td>29.87</td><td>27.19</td><td>24.32</td><td>21.43</td><td>26.49</td><td>30.78</td><td>23.62</td></tr>
<tr><td>6</td><td>22</td><td>21.72</td><td>21.18</td><td>17.55</td><td>16.39</td><td>15.91</td><td>19.77</td><td>20.63</td><td>16.65</td></tr>
<tr><td>7</td><td>7</td><td>14.57</td><td>15.02</td><td>11.49</td><td>11.39</td><td>13.20</td><td>14.75</td><td>13.40</td><td>12.19</td></tr>
<tr><td>8</td><td>9</td><td>9.77</td><td>10.65</td><td>7.61</td><td>8.08</td><td>9.05</td><td>11.01</td><td>9.48</td><td>9.16</td></tr>
<tr><td>9</td><td>5</td><td>6.56</td><td>7.55</td><td rowspan="2">8.58</td><td rowspan="2">10.04</td><td>6.78</td><td>8.22</td><td rowspan="2">10.99</td><td rowspan="2">12.33</td></tr>
<tr><td>10</td><td>3</td><td rowspan="2">7.35</td><td>5.36</td><td>5.26</td><td>6.13</td></tr>
<tr><td>11</td><td>6</td><td rowspan="2">6.49</td><td rowspan="5">13.18</td><td rowspan="2">5.33</td><td rowspan="2">8.32</td><td rowspan="2">7.99</td><td rowspan="5">11.26</td><td rowspan="2">7.32</td></tr>
<tr><td>12</td><td>6</td><td rowspan="4">6.01</td></tr>
<tr><td>13</td><td>1</td><td rowspan="3">6.56</td><td rowspan="3">6.24</td><td rowspan="2">5.39</td><td rowspan="3">10.04</td><td rowspan="3">11.00</td></tr>
<tr><td>14</td><td>3</td></tr>
<tr><td>15+</td><td>8</td><td>5.24</td></tr>
<tr><td>Total</td><td>486</td><td>486</td><td>486</td><td>486</td><td>486</td><td>486</td><td>486</td><td>486</td><td>486</td></tr>
<tr><td colspan="2">χ^2</td><td>50.29</td><td>27.54</td><td>19.43</td><td>10.14</td><td>11.09</td><td>49.07</td><td>34.79</td><td>10.38</td></tr>
<tr><td colspan="2">d.f.</td><td>9</td><td>9</td><td>7</td><td>7</td><td>5</td><td>9</td><td>7</td><td>9</td></tr>
</table>

Table 3: Observed and expected distribution of number of menstrual cycles required for the first conception (smoker females)

<table>
<tr><th rowspan="3">No. of menstrual cycle</th><th rowspan="3">Observed frequency</th><th colspan="4">Expected frequency
(ML Method)</th><th colspan="3">Expected frequency
MM Method</th></tr>
<tr></tr>
<tr><th>Model-I*</th><th>Model-II</th><th>Model-III</th><th>Model-V**</th><th>Model-II</th><th>Model-III</th><th>Model-IV***</th></tr>
<tr><td>1</td><td>29</td><td>22.73</td><td>29.00</td><td>23.96</td><td>29.00</td><td>28.91</td><td>23.24</td><td>28.27</td></tr>
<tr><td>2</td><td>16</td><td>17.56</td><td>14.83</td><td>18.09</td><td>16.70</td><td>14.87</td><td>17.72</td><td>15.58</td></tr>
<tr><td>3</td><td>17</td><td>13.57</td><td>11.73</td><td>13.69</td><td>11.28</td><td>11.76</td><td>13.54</td><td>11.79</td></tr>
<tr><td>4</td><td>4</td><td>10.49</td><td>9.28</td><td>10.38</td><td>8.43</td><td>9.30</td><td>10.37</td><td>9.28</td></tr>
<tr><td>5</td><td>3</td><td>8.10</td><td>7.34</td><td>7.10</td><td>7.23</td><td>7.35</td><td>7.05</td><td>7.34</td></tr>
<tr><td>6</td><td>9</td><td>6.26</td><td>5.81</td><td>6.01</td><td>6.34</td><td>5.82</td><td>6.11</td><td>5.80</td></tr>
<tr><td>7</td><td>4</td><td rowspan="2">8.58</td><td rowspan="2">8.23</td><td>4.59</td><td>5.01</td><td rowspan="2">8.24</td><td rowspan="3">11.15</td><td rowspan="2">8.22</td></tr>
<tr><td>8</td><td>5</td><td rowspan="2">6.20</td><td rowspan="2">6.45</td></tr>
<tr><td>9</td><td>1</td><td rowspan="3">6.85</td><td rowspan="3">8.38</td><td rowspan="4">8.38</td><td rowspan="6">13.72</td></tr>
<tr><td>10</td><td>1</td><td rowspan="4">5.83</td><td rowspan="5">9.56</td><td rowspan="4">6.19</td></tr>
<tr><td>11</td><td>1</td></tr>
<tr><td>12</td><td>3</td><td rowspan="3">5.87</td><td rowspan="3">5.40</td></tr>
<tr><td>13</td><td>2</td><td rowspan="2">5.38</td></tr>
<tr><td>14+</td><td>5</td><td>4.14</td><td>4.62</td></tr>
<tr><td>Total</td><td>100</td><td>100</td><td>100</td><td>100</td><td>100</td><td>100</td><td>100</td><td>100</td></tr>
<tr><td colspan="2">χ^2</td><td>16.26</td><td>11.01</td><td>10.37</td><td>9.71</td><td>10.99</td><td>10.34</td><td>9.77</td></tr>
<tr><td colspan="2">d.f.</td><td>5</td><td>6</td><td>7</td><td>3</td><td>6</td><td>6</td><td>4</td></tr>
</table>

Table 4: Estimates of parameters involved in the models

Models	Parameter Estimated	Estimates for all females		Estimates for non-smoker females		Estimates for smoker females	
		MM Method	ML Method	MM Method	ML Method	MM Method	ML Method
Homogeneous Model	p	0.31	0.31	0.33	0.33	0.21	0.21
Inflated Model	p	0.24	0.27	0.25	0.29	0.21	0.21
	α	0.72	0.84	0.69	0.84	0.89	0.89
Geometric-Beta Distribution	a	22.89	19.78	19.73	12.67	35.12	33.67
	b	49.67	40.72	38.16	10.89	116.01	106.86
Mixture Model	α	0.56	0.59	0.48	0.51	0.89	–
	p	0.22	0.25	0.22	0.26	0.21	–
	$1-\alpha$	0.44	0.41	0.52	0.49	0.11	–
	θ	0.59	0.58	0.57	0.56	0.92	–
Model-V	α	–	0.69	–	0.72	–	0.92
	a_1	–	21.23	–	27.89	–	21.23
	b_1	–	45.67	–	68.90	–	55.67
	$1-\alpha$	–	0.31	–	0.28	–	0.08
	a_2	–	48.91	–	56.90	–	80.94
	b_2	–	60.84	–	80.83	–	21.91