

Research Article

Estimation of Probability of Coition on Different Days of a Menstrual Cycle near the Day of Ovulation: An Application of Theory of Markov Chain

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Abstract

Various research studies have evidences that the day of coition with respect to the day of ovulation in a menstrual cycle may affect the probability of conception as well as the sex ratio at birth. But the data on coition probabilities on different days of the menstrual cycle is rarely available and in this situation theoretical estimation may be helpful for estimating these probabilities. This research work presents a new theoretical approach for estimating the probabilities of coition near the day of ovulation for different couple coital patterns. These probabilities are then utilized for estimating the probability of conception in a menstrual cycle and the sex ratio at birth.

Introduction

In humans, normally there are 6 days in one menstrual cycle during which coition can result in to a conception (the 5 days before ovulation and the day of ovulation itself) and woman becomes pregnant (Wilcox et al., 1995; Dunson et al., 1999; Wilcox et al., 2004). This 6 day window is referred to as the 'most fertile period' in one menstrual cycle. Here the term 'ovulation' refers to the phase of a female's menstrual cycle in which a partially mature ovum is released from the ovarian follicles into the oviduct or the fallopian tube. After ovulation, during the luteal phase the egg is available to be fertilized by sperm. In humans ovulation normally occurs somewhere between the 10th and 19th day of a menstrual cycle. Many researchers have studied about the probability of conception on different days or phases of menstrual cycle (Barrett and Marshall, 1969; Royston, 1982; Royston, 1991; Dunson et al., 1999; Dunson et al., 2001; Stanford et al., 2002). Some studies indicate that the highest probability of clinically evident conception occurs with coition 1 or 2 days before ovulation, rather than the day of ovulation itself (Barrett and Marshall, 1969; Dunson et al., 1999; Dunson et al., 2001; Stanford et al., 2002). On the other hand, Wilcox et al. (1995) conjectured about highest probability of conception on the day of ovulation

In addition to above, it is argued in many studies that the sex of the child is affected by the day of coition in relation to the day of ovulation (Guerrero, 1974; James, 1976; Harlap, 1979; France et al., 1984; Perez et al., 1985; France et al., 1992; James, 1997; James, 1999; James, 2000). Guerrero (1974) found that both in natural and artificial insemination, sex ratio changes significantly during the menstrual interval. He obtained a more or less U-shaped distribution of the probability that the sex of child will be male for coition taking place on different days in relation to ovulation. James (1976; 1997; 1999; 2000) also found the existence of correlation between time of fertilization within menstrual cycle and the sex ratio of offspring. Harlap (1979) and Perez et al. (1985) in their studies have observed the variations in sex ratio for inseminations occurring on different days of the menstrual cycle. France et al. (1984; 1992) have found male preferring birth sex ratio when coition preceded ovulation by 2 or more days. Although the association found by France et al. (1984; 1992) is statistically significant, they are skeptical about the observed association since number of pregnancies

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involved in their data is too small. On the other hand, Wilcox et al. (1995) and Gray et al. (1998) have argued in their studies that timing of insemination does not affect the sex ratio that present contradiction to studies mentioned earlier. This suggests that the pattern of coition during a menstrual cycle may affect the sex ratio at birth as well as chance of conception in a menstrual cycle. Thus it seems desirable to examine this pattern. Some studies have tried to find the probability of coition on different days of the menstrual cycle relative to ovulation and also the average number of coitions in a week (Harvey, 1987; Wilcox et al., 2004).

However, all these studies are based on prospective data on coition on different days relative to ovulation in a menstrual cycle and contain a small sample of women who agreed to become an object for the study. In fact, such data for general populations are not available in literature especially due to difficulty in determining the exact day of ovulation of a female in the surveyed population.

The objective of the present study is to make an attempt towards theoretically estimating the probability of coition on different days (especially near the day of ovulation) of a menstrual cycle under different assumed patterns of coition during a menstrual cycle. For this purpose the theory of Markov chain has been utilized. These estimated probabilities are then used to find the probability of conception in one menstrual cycle and sex ratio at birth for various patterns of coition in a menstrual cycle.

Brief Overview of the Theory of Markov Chains

In the theory of Markov chains, we consider the simplest generalization of independent trials which consists in permitting the outcome of any trial to depend upon the outcome of the directly preceding trial. The possible out- comes of a trial are usually referred to as the possible 'states' of the Markov chain. Here it is assumed that the trials are performed at a uniform rate so that the number of the trial serves as the time parameter. The outcome (E_k) in any trial is no longer associated with a fixed probability (p_k) but to every pair of the states ($E_j ; E_k$) of the two successive trials (say n^{th} and $(n + 1)^{\text{st}}$ trials), there corresponds a conditional probability p_{jk} . Here p_{jk} is called the probability of transition from the state E_j at n^{th} trial to state E_k at $(n+1)^{\text{st}}$ trial. Hence, given that E_j has occurred at some trial, the probability of occurrence of E_k at the next trial is p_{jk} .

In more general case, m -step transition probability, i.e., the probability of transition from state E_j at n^{th} trial to the state E_k at $(n + m)^{\text{th}}$ trial is denoted by $p_{jk}^{(m)}$. In fact,

$$p_{jk}^{(m)} = \sum_v p_{jv} p_{vk}^{m-1}$$

Thus $p_{jk} = p_{jk}^{(1)}$ is also called the one-step transition probability. Transition probabilities p_{jk} satisfy,

$$p_{jk} \geq 0, \sum_k p_{jk} = 1 \forall j$$

Arranging these probabilities in the matrix form gives the 'transition probability matrix' (t.p.m.) of the Markov chain. For convenience the states E_j, E_k etc. can be represented as j, k and so on. Now suppose that a system starts with the state j . Let $f_{jk}^{(n)}$ be the probability that it reaches the state k for the first time at the n^{th} step and $p_{jk}^{(n)}$ is the probability that it reaches the state k at the n^{th} step (not necessarily for the first time). Also let F_{jk} be the probability that starting with state j , the system will ever reach k , i.e.,

$$F_{jk} = \sum_{n=1}^{\infty} f_{jk}^{(n)}$$

When $F_{jk} = 1$, it is certain that starting with state j , system will reach state k and in this case is a proper probability distribution and this gives the first passage time distribution for k given that system starts with j . So the mean first passage time from the state j to the state k is given by,

$$\mu_{jk} = \sum_{n=1}^{\infty} n \cdot f_{jk}^{(n)}$$

In particular when $k=j$, $\{f_{jj}^{(n)}, n=1, 2, \dots\}$ represents the distribution of the recurrence time of j and $F_{jj} = \sum_{n=1}^{\infty} n \cdot f_{jj}^{(n)} = 1$ will imply that the return to state j is certain. In this case,

$$\mu_{jj} = \sum_{n=1}^{\infty} n \cdot f_{jj}^{(n)}$$

is known as the 'mean recurrence time' for the state j .

In fact for studying any Markov chain and its properties, the specification of its states, the matrix of transition probabilities (P) and the probability distribution $\{s_k\}$ are required to be specified, where s_k is the probability that the system starts with the state E_k at the initial trial. Now an attempt has been made to apply this theory of Markov chains to compute the probability of coition on different days of a menstrual cycle especially near the day of ovulation. Here each day of the menstrual cycle is considered as a trial and each trial has two possible outcomes: either a coition occurs on that day or coition does not occur on that day.

For the purpose, it is required to specify the states and transition probability matrix. Consider that E_k represents the state that the last coition took place k days ($k = 0, 1, 2, \dots$) before the specified day. Obviously E_0 represents the state that the coition takes place on the specified day, E_1 represents the state that the last coition took place one day before the specified day and so on. After the specification of states, it is required to specify the transition probability matrix. In this context, it is important to note that on any day there are only two possibilities: either a coition will occur or not. Thus if the system is in state E_k on a particular day, then on the next day it will either be in state E_0 (if the coition takes place on that day) or will be in the state E_{k+1} (if the coition does not take place on that day). So from any state E_k , the transitions are possible only to either E_0 or E_{k+1} and transition probabilities to all other states will be zero.

Let s_j stands for the probability of the state E_j ($j = 0, 1, 2, \dots$) at the initial trial. The unconditional probability of entering E_k at the n^{th} step is then,

$$s_k^{(n)} = \sum_j s_j p_{jk}^{(n)}$$

In many situations it is observed that the influence of the initial state gradually wears off, so that for large n , the distribution in the above equation becomes nearly independent of the initial distribution $\{s_j\}$. This is the case if $p_{jk}^{(n)}$ converges to a limit independent of j , that is if the n^{th} power of the transition probability matrix (P^n) converges to a matrix with identical rows.

Method and Application

Let us assume that the initial trial starts with the day on which the first coition occurs after the start of menstrual cycle of woman. It is already mentioned in the previous section that E_k represents the state that the last coition occurred k days before the considered day, $k = 0, 1, 2, \dots$. In the context of application, the maximum possible value of k is to be specified. The maximum possible value of k implies that the number of days between two consecutive coitions cannot exceed this maximum possible value of k . Obviously this value of k will depend upon the pattern of coition during a menstrual cycle but for application purpose we take the maximum possible value of k to be 5 implying that after the day of a coition the next coition has to occur during the next 5 days. This may nearly be true especially in the case of younger females (less than 30 years of age) in most of the populations. Of course larger values of k may also be taken for females of higher ages.

Further, the transition probabilities in the transition probability matrix P will heavily depend upon the pattern of coition, but it may reasonably be assumed that p_{k0} increases as k increases (It has already been argued earlier that only p_{k0} and $p_{k:k+1}$ can be non-zero). For illustration, such a matrix P_1 satisfying the above properties is considered, which is given below,

$$P_1 = \begin{matrix} & E_0 & E_1 & E_2 & E_3 & E_4 & E_5 \\ \begin{matrix} E_0 \\ E_1 \\ E_2 \\ E_3 \\ E_4 \\ E_5 \end{matrix} & \begin{pmatrix} 0.2 & 0.8 & 0 & 0 & 0 & 0 \\ 0.3 & 0 & 0.7 & 0 & 0 & 0 \\ 0.4 & 0 & 0 & 0.6 & 0 & 0 \\ 0.6 & 0 & 0 & 0 & 0.4 & 0 \\ 0.8 & 0 & 0 & 0 & 0 & 0.2 \\ 1.0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{matrix}$$

It is known that normally the ovulation takes place between 10th to 19th days of menstrual cycle and so, on an average 15th day of the menstrual cycle can be considered to be the day of ovulation. If it is assumed that first coition in a menstrual cycle occurs 3-4 days after the start of menstruation, then the most probable day of ovulation may be considered to be around 12th day from the initial day of first coition in the menstrual cycle i.e. from initial trial in the theory of Markov chain. The sexual behavior near this day may be obtained by taking the 12th power of matrix P_1 . The computation of 10th, 11th and 12th powers of the matrix P_1 is done and the obtained matrices are given below,

From the matrices p_1^{10} , p_1^{11} and p_1^{12} it is seen that the rows in these matrices are almost identical indicating that the effect of initial trial has wearied off. Consequently, it can be safely assumed that the probabilities of being in the states E_0, E_1, \dots, E_5 near the day of ovulation have become stable. Thus the probabilities in the rows of matrix p_1^{12} represent that after 12 days from the day of first coition of the menstrual cycle, around 35% couples will be in state E_0 , showing that 35 percent females will have coition on the day of ovulation (if 12th day is considered as the day of ovulation), around 28% couples will be in state E_1 i.e. there will be 28% couples whose last coition has taken place one day before the day of ovulation, around 20% will be in state E_2 and so on. Thus if the 12th day after the initiation of first coition in the cycle is considered to be the day of ovulation, then this matrix (p_1^{12}) provides the probability distribution of pattern of coition near the day of ovulation.

Further, since 10th and 11th powers of P_1 are also almost identical to 12th power of P_1 , hence even if the ovulation takes place on 10th or 11th day from the day of first coition, the results will be the same as discussed above. Now from the matrix P_1 , the distribution of first passage time from the state E_0 to the state E_0 , $\{f_{00}^{(n)}, n=1, 2, \dots\}$, can be obtained as follows: Starting with the state E_0 , the probability of transition to the state E_0 in one step will be given by the first element in the first cell of P_1 , probability of transition to the state E_0 in two steps will be the product of transition probabilities corresponding to the transitions $0 \rightarrow 1$ and $1 \rightarrow 0$. Similarly probability of transition to the state E_0 in 3 steps will be the product of transition probabilities for the transitions $0 \rightarrow 1$ and $1 \rightarrow 2$ and $2 \rightarrow 0$. Similarly other probabilities can be found. Thus the probability distribution of the first passage time (recurrence time) for state E_0 is obtained which is shown in Table 1.

The probability distribution obtained in Table 1 is nothing but the probability distribution of transition from state E_0 to E_0 for the first time in n steps. Here it can be observed that probability that a couple will be in state E_0 after two-steps is maximum (0.24) i.e. for 24% couples last coition takes place two days before the considered day (here the 12th day after the initiation of first coition in a menstrual cycle or the day of ovulation).

$$P_1^{10} = \begin{matrix} & E_0 & E_1 & E_2 & E_3 & E_4 & E_5 \\ \begin{matrix} E_0 \\ E_1 \\ E_2 \\ E_3 \\ E_4 \\ E_5 \end{matrix} & \begin{pmatrix} 0.35 & 0.28 & 0.20 & 0.12 & 0.05 & 0.01 \\ 0.35 & 0.28 & 0.20 & 0.12 & 0.05 & 0.01 \\ 0.35 & 0.28 & 0.20 & 0.12 & 0.05 & 0.01 \\ 0.35 & 0.28 & 0.19 & 0.12 & 0.05 & 0.01 \\ 0.35 & 0.28 & 0.19 & 0.12 & 0.05 & 0.01 \\ 0.35 & 0.28 & 0.19 & 0.12 & 0.05 & 0.01 \end{pmatrix} \end{matrix}$$

$$P_1^{11} = \begin{matrix} & E_0 & E_1 & E_2 & E_3 & E_4 & E_5 \\ \begin{matrix} E_0 \\ E_1 \\ E_2 \\ E_3 \\ E_4 \\ E_5 \end{matrix} & \begin{pmatrix} 0.35 & 0.28 & 0.20 & 0.12 & 0.05 & 0.01 \\ 0.35 & 0.28 & 0.20 & 0.12 & 0.05 & 0.01 \\ 0.35 & 0.28 & 0.19 & 0.12 & 0.05 & 0.01 \\ 0.35 & 0.28 & 0.20 & 0.12 & 0.05 & 0.01 \\ 0.35 & 0.28 & 0.19 & 0.12 & 0.05 & 0.01 \\ 0.35 & 0.28 & 0.19 & 0.12 & 0.05 & 0.01 \end{pmatrix} \end{matrix}$$

$$P_1^{12} = \begin{matrix} & E_0 & E_1 & E_2 & E_3 & E_4 & E_5 \\ \begin{matrix} E_0 \\ E_1 \\ E_2 \\ E_3 \\ E_4 \\ E_5 \end{matrix} & \begin{pmatrix} 0.35 & 0.28 & 0.20 & 0.12 & 0.05 & 0.01 \\ 0.35 & 0.28 & 0.20 & 0.12 & 0.05 & 0.01 \\ 0.35 & 0.28 & 0.20 & 0.12 & 0.05 & 0.01 \\ 0.35 & 0.28 & 0.20 & 0.12 & 0.05 & 0.01 \\ 0.35 & 0.28 & 0.19 & 0.12 & 0.05 & 0.01 \\ 0.35 & 0.28 & 0.19 & 0.12 & 0.05 & 0.01 \end{pmatrix} \end{matrix}$$

Now from this probability distribution the mean recurrence time $\mu_{00} = \sum_{n=1}^{\infty} n.f_{00}^{(n)}$ can be calculated and it is obtained as 2.86 days. It represents that on an average couples will take around 2.86 days to go for the next coition. Therefore, mean number of coitions per week will be $(7/2.86)=2.45$. Similarly, some other hypothetical transition probability matrices P_2 , P_3 , P_4 and P_5 are considered to see the changes in mean recurrence times for various hypothetical transition patterns. Considered matrices are as follows:

$$P_2 = \begin{matrix} & E_0 & E_1 & E_2 & E_3 & E_4 & E_5 \\ \begin{matrix} E_0 \\ E_1 \\ E_2 \\ E_3 \\ E_4 \\ E_5 \end{matrix} & \begin{pmatrix} 0.1 & 0.9 & 0 & 0 & 0 & 0 \\ 0.2 & 0 & 0.8 & 0 & 0 & 0 \\ 0.5 & 0 & 0 & 0.5 & 0 & 0 \\ 0.8 & 0 & 0 & 0 & 0.2 & 0 \\ 0.9 & 0 & 0 & 0 & 0 & 0.1 \\ 1.0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{matrix}$$

$$P_4 = \begin{matrix} & E_0 & E_1 & E_2 & E_3 & E_4 & E_5 \\ \begin{matrix} E_0 \\ E_1 \\ E_2 \\ E_3 \\ E_4 \\ E_5 \end{matrix} & \begin{pmatrix} 0.10 & 0.90 & 0 & 0 & 0 & 0 \\ 0.15 & 0 & 0.85 & 0 & 0 & 0 \\ 0.45 & 0 & 0 & 0.55 & 0 & 0 \\ 0.65 & 0 & 0 & 0 & 0.35 & 0 \\ 0.85 & 0 & 0 & 0 & 0 & 0.15 \\ 1.0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{matrix}$$

$$P_3 = \begin{matrix} & E_0 & E_1 & E_2 & E_3 & E_4 & E_5 \\ \begin{matrix} E_0 \\ E_1 \\ E_2 \\ E_3 \\ E_4 \\ E_5 \end{matrix} & \begin{pmatrix} 0.15 & 0.85 & 0 & 0 & 0 & 0 \\ 0.25 & 0 & 0.75 & 0 & 0 & 0 \\ 0.6 & 0 & 0 & 0.4 & 0 & 0 \\ 0.7 & 0 & 0 & 0 & 0.3 & 0 \\ 0.8 & 0 & 0 & 0 & 0 & 0.2 \\ 1.0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{matrix}$$

$$P_5 = \begin{matrix} & E_0 & E_1 & E_2 & E_3 & E_4 & E_5 \\ \begin{matrix} E_0 \\ E_1 \\ E_2 \\ E_3 \\ E_4 \\ E_5 \end{matrix} & \begin{pmatrix} 0.15 & 0.85 & 0 & 0 & 0 & 0 \\ 0.2 & 0 & 0.8 & 0 & 0 & 0 \\ 0.4 & 0 & 0 & 0.6 & 0 & 0 \\ 0.5 & 0 & 0 & 0 & 0.5 & 0 \\ 0.7 & 0 & 0 & 0 & 0 & 0.3 \\ 1.0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{matrix}$$

It is to be mentioned here that in all the matrices mentioned above probability of coition on a particular day is increasing as the duration from the previous coition increases (see the first column of the matrices). Again it is observed that 10th and higher powers of the matrices P_2 , P_3 , P_4 and P_5 comes out to be the matrices with identical rows. Table 2 shows the identical rows of the hypothesized matrices P_2 , P_3 , P_4 and P_5 .

The rows in the Table 2 are nothing but the probability distribution that couples will be in state E_0 , E_1 , E_2 , E_3 , E_4 and E_5 after 10-12 days of initial point or at the day of ovulation. Now from the matrices P_2 , P_3 , P_4 and P_5 , the distribution of first passage time for the state E_0 and mean recurrence times are obtained which are shown in the Table 3.

All the matrices P_1 , P_2 , P_3 , P_4 and P_5 are hypothetical but looking at the calculated values of mean number of coitions per week among couples following the assumed pattern of coition, it can be observed that values of mean number of coitions per week are in the range of 2.15 to 2.47 which seem to be close to reality. Many previous studies have observed mean number of coitions per week near this range. Wilcox et al. (2004) have reported the mean frequency of intercourse per day as 0.29 which is equivalent to almost twice a week. Kinsey (1953) found that among white USA women, teenage brides had intercourse on an average 3.7 times a week which dropped slightly to 2.6 by thirty, 2.1 at forty and 1.5 at fifty years of age. Anthropologist Moni Nag (1972) has compared data on coital frequency in Muslim and Hindu groups from three West Bengal villages with the findings of Kinsey for white married women in USA. Findings reveal that Hindu women of age group 20- 29 have on an average 1.9 coitions per week while Sheikh and Non- Sheikh Muslim women of the same age group have around 2.4 and 2.6 coitions per week respectively (For details see (Potts and Selman, 1979, p. 75)). Yadava and Rai (1989) have also reported similar findings. Thus, the considered patterns of coition seem to be somewhat close to reality.

Possible Impact of 'Coital Pattern' on Sex Ratio at Birth

The previous section was focused towards finding the probability distribution of day of last coition near the day of ovulation in a menstrual cycle utilizing the theory of Markov chains. This was mainly done to examine the possible impact of sexual pattern on sex ratio at birth as well as chance of conception in a menstrual cycle. It has already been mentioned that various researchers have argued that the chance of conception as well as the sex of the child are affected by the day of coition in relation to the day of ovulation but the results are not conclusive. However, whatever be these conclusions, if the probabilities of conception due to coition occurring on different days near the day of ovulation are known, the probability of conception in a menstrual cycle can be computed easily under different assumed patterns of coition (transition probability matrices P_1 , P_2 , P_3 etc.).

Now let a_k is the probability that the system will be in state E_k on the day of ovulation and b_k is the probability of conception from coition occurring on day $-k$ ($k = 0, 1, 2, 3, 4, 5$), then probability of conception in one menstrual cycle would be,

$$PC = \sum_k a_k \cdot b_k \quad (1)$$

Table 4 presents the estimate of probability of conception in one menstrual cycle under assumed coital patterns. Here it is assumed that the probabilities of conception from last coition on days 0, -1, -2, -3, -4, -5 (the day of ovulation taken as '0') i.e. b_k are approximately 0.3, 0.3, 0.25, 0.15, 0.15, 0.1 respectively (Wilcox et al., 1995). But it is observed in Wilcox et al. (1995) that around 50 percent of the conceptions result in pregnancy loss. In this case, under the coital patterns of matrices P_1 , P_2 , P_3 , P_4 and P_5 the probability of birth in one menstrual cycle will be around 0.13. A detailed discussion on pregnancy loss is also given in Potts and Selman (1979), pp.19-21.

Now if c_k is the probability of a male birth for conception occurring due to coition on day $-k$ ($k = 0, 1, 2, 3, 4, 5$), then sex ratio at birth will be,

$$SRB = \frac{\sum_k a_k b_k c_k}{PC} \quad (2)$$

Guerrero (1974) has obtained probabilities of male birth for conceptions occurring due to coition on days 0, -1, -2, -3, -4, -5 i.e. c_k as 0.43, 0.45, 0.51, 0.52, 0.60, 0.68 respectively. In addition, we have also calculated the probabilities of male birth for conceptions occurring on days 0, -1, -2, -3, -4 i.e. c_k as 0.47, 0.49, 0.60, 0.58, 0.58 respectively using the figure 4 in Wilcox et al. (1995), (Note that in Wilcox et al. (1995) probabilities c_k for $k=3$ and 4 are based on very small number of women, so we have clubbed the two group of observations and considered the value of c_3 and c_4 as 0.58). Table 5 shows the obtained sex ratio at birth for assumed coital patterns.

Discussion

The present paper presents a new approach for theoretical estimation of the probabilities of coition on different days of a menstrual cycle of a woman which may be helpful in estimating the probability of conception in a menstrual cycle as well as the sex ratio at birth in the population. Although the issue of probability of conception on different days near the day of ovulation as well as its impact on sex of the baby are yet not fully resolved, still if these are known, the present paper would be helpful in throwing new lights on these aspects.

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Table 1: Distribution of the 'First Passage Time' for the State E_0

n	$f_{00}^{(n)}$ *
1	0.200
2	0.240
3	0.224
4	0.202
5	0.108
6	0.027

*Distribution of first passage time from state E_0 to state E_0

Table 2: Entries of the identical rows for the 10th and higher powers of the matrices P_2 , P_3 , P_4 and P_5

P_2	0.33	0.30	0.24	0.12	0.02	0.002
P_3	0.35	0.30	0.23	0.09	0.03	0.01
P_4	0.31	0.28	0.24	0.13	0.05	0.01
P_5	0.31	0.27	0.21	0.13	0.06	0.02

Table 3: Distributions of the 'First Passage Time' and 'Mean Recurrence Times' for the State E_0 for the Transition Probability Matrices P_2 , P_3 , P_4 and P_5

n	$f_{00}^{(n)}$ * for			
	P_2	P_3	P_4	P_5
1	0.10	0.15	0.10	0.15
2	0.18	0.21	0.14	0.17
3	0.36	0.38	0.34	0.27
4	0.29	0.18	0.27	0.20
5	0.06	0.06	0.13	0.14
6	0.01	0.02	0.02	0.06
Mean Recurrence Time	3.06	2.83	3.26	3.20
Mean number of coitions per week	2.29	2.47	2.15	2.19

*Distribution of first passage time from state E_0 to state E_0

Table 4: Probabilities of Conception in One Menstrual Cycle under Assumed Coital Patterns

	PC*
1	0.200
2	0.240
3	0.224
4	0.202
5	0.108
6	0.027

*Probability of conception in one menstrual cycle

Table 5: Sex ratio at birth under assumed coital patterns

	SRB_G^a	SRB_W^b
P ₁	0.463	0.512
P ₂	0.462	0.514
P ₃	0.462	0.512
P ₄	0.467	0.517
P ₅	0.467	0.515

^aSRB calculated for c_k of Guerrero (1974), ^bSRB calculated for c_k of Wilcox et al. (1995)