

## Dynamics of COVID-19 Cases in India: A Statistical Model

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**Abstract:** The COVID-19 cases and related deaths have been increasing throughout the world, without any let up. Health planners, medical professionals, researchers, and scientists have been working relentlessly in containing the disease from the first day, with hardly any success. Hence, study has been under taken with an objective to establish the statistical model of trends in COVID-19 active cases for states of India, so that disease containment performance is assessed objectively, taking model as reference level. The state-wise data from Kaggle website were used to study the case load, cure rate, and levels and trends in active cases by Modified Second Degree Polynomial Model in Indian state using SPSS-22 and Microsoft Office-10. Spot snap of COVID-19 situation as on 21<sup>st</sup> June, 2020 has also been depicted to fill up the gap infirmity. The COVID-19 Active Case Rate varied from 0 to 139 per 100,000 populations. COVID-19 Active Case Rate was highest in Delhi followed by Maharashtra, with best state Meghalaya (COVID-19 with Active case rate = 0.3) and Andaman and Nicobar (2.9). In addition, the model fitted well with Coefficient of Determinants ( $R^2$ ) as highest ( $R^2=0.999$ ), with more than 0.80  $R^2$  in 23 states, the states with  $R^2$  less than 0.80 had very less number of COVID-19 cases, hence, model did not fit well. The Modified Quadratic Model had the best fit, to assess the performance of states in controlling the COVID-19. The models are expected to be used by Health Planners and Health Professionals, as the models were fitted well in 23 states. Further, the reasons for wide variation in active cases of COVID-19 may be unrest in community due to labour movement. Hence, for containment of COVID-19, the need is to restrict labour movement, and pay attention on enhancement of the health services along with provisioning of public necessities.

**Keywords:** COVID-19, Inactive Case load, Case Load, States of India, Modified Second Degree Polynomial

### Introduction

The outbreak of novel coronavirus disease 2019 (COVID-19) resulted in more than 7,75,000 cases along with 36,000 deaths in more than 160 countries by March 30, 2020 (WHO, 2020; Dong et al., 2020). COVID-19 cases were first detected in Wuhan, China, in December, 2019. Initial outbreak of maximum cases, internationally informed to have history of travel to Wuhan (China) (nCoV-2019 Data Working Group, 2020). Disease COVID-19 spreads from men to men, hence, estimation of fluctuations in transmission over time provide vision in epidemiological aspects of the disease (Camacho et al., 2015). Furthermore, statistical models in COVID-19 spread along with trends in disease will help in understanding outbreak control measures (Riley et al., 2003; Funk et al., 2017). Models and trends in COVID-19 predict possible outbreak of the disease (Viboud et al., 2018), Model may also be help all other nations to control and monitor the disease (Cooper et al., 2006).

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Modelling approaches helps in monitoring delays in case-control and case management (Nishiura et al., 2009; Aylward et al., 2014). Additionally, individual data sources might be incomplete, biased, or may only capture certain facts of the outbreak and its dynamics. Appropriate mathematical model captures the average nature, control and management of COVID-19 transmission across countries.

To ease the burden on the healthcare system and providing the best possible care for COVID-19 patients, appropriate models are expected to help in estimation of the risk of infection for better planning, monitoring and control measures. Theoretically, each disease, more so acute and communicable, is expected to follow a pattern, usually, i) Acceleratory phase, ii) Optimum level and ii) De-acceleratory phase. Usually, at the time of evaluation, the time, when data is available, disease is expected to be in de-acceleratory phase. Hence, an attempt has been made to study the levels and trends of the COVID-19 active cases in the states of India. So that necessary action as lockdown, de-lockdown, standardization of line of treatment providing necessities, and enforcement of law are taken at appropriate time, to enhance health and economic benefits of the community.

### Materials and Methods

The state-wise data of COVID-19 as on 1<sup>st</sup> May, 2020 were downloaded from website Kaggle (COVID-19 in India, 2020), the platform for extracting the data on novel corona virus diseases 2019, to help better planning and decision. Further, spot snap of COVID-19 situation as on 21<sup>st</sup> June, 2020 has also been depicted to fill up the gap in firmity. Active Cases, in Indian states followed Modified Second-Degree Polynomial as is expected too:

$$\text{Active Cases} = a e^{b_1 t + b_2 t^2}$$

Where,  $a = e^{b_0}$

$b_0, b_1, b_2$  and are the Regression Coefficients.

Active Case Ratio (ACR) per week for estimated Active Cases has been computed as

$$\text{ACR} = \frac{EAC_{i+7}}{EAC_i}$$

Where,  $EAC_{i+7}$  - Estimated Active Cases at  $(i+7)^{\text{th}}$  day and  
 $EAC_i$  - Estimated Active Cases at  $i^{\text{th}}$  day

Seven days Active Case Ratio, at place of one day has been computed to get magnifiable change and to minimising the misreporting. Active Case Ratio is expected to take values greater or equal to zero; '0' indicates no case of COVID-19, '1' peak level of COVID-19, and more than '1' increasing, as Growth Ratio increases. The 95% Confidence Interval (CI) for Active Case Ratio per week has been computed as:

$$95\% \text{ CI} = e^{\left( \log_e(\text{Active Case Ratio}) \pm 1.96 \sqrt{\frac{1}{AC_{i+7}} + \frac{1}{AC_i}} \right)}$$

Where,  $AC_{i+7}$  - Observed Active Case at  $(i+7)^{\text{th}}$  time and  $AC_i$  - Observed Active Cases at  $i^{\text{th}}$  time

$$\text{COVID - 19 Rate} = \frac{\text{Active Cases}}{\text{Population}} \times 100,000$$

$$\text{Case Fatality Rate} = \frac{\text{Deaths}}{\text{Cured} + \text{Deaths}} \times 1,000$$

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$$\text{Active Case Load} = \frac{\text{Active Cases}}{\text{Cured} + \text{Deaths} + \text{Active Cases}} \times 100$$
$$\text{COVID - 19 Inactive Case Load} = \frac{\text{Cured} + \text{Deaths}}{\text{Cured} + \text{Deaths} + \text{Active Cases}} \times 100$$

Modified Second-Degree Polynomial using Partial Least Square (PLS) Regression Procedure has been used, so that Health Planners, Health Professional can use, the information to assess their performance in containing the disease in comparative perspective. Furthermore, the COVID-19 disease and its fatality rates have been presented as on 21 June, 2020 to cover the gap of unavailability of information. All the available models including Exponential have been tried. The best fit model as per Coefficient of Determination and Visual depiction of observed trends of Active Cases of COVID-19, Modified Second-Degree Polynomial has been selected.

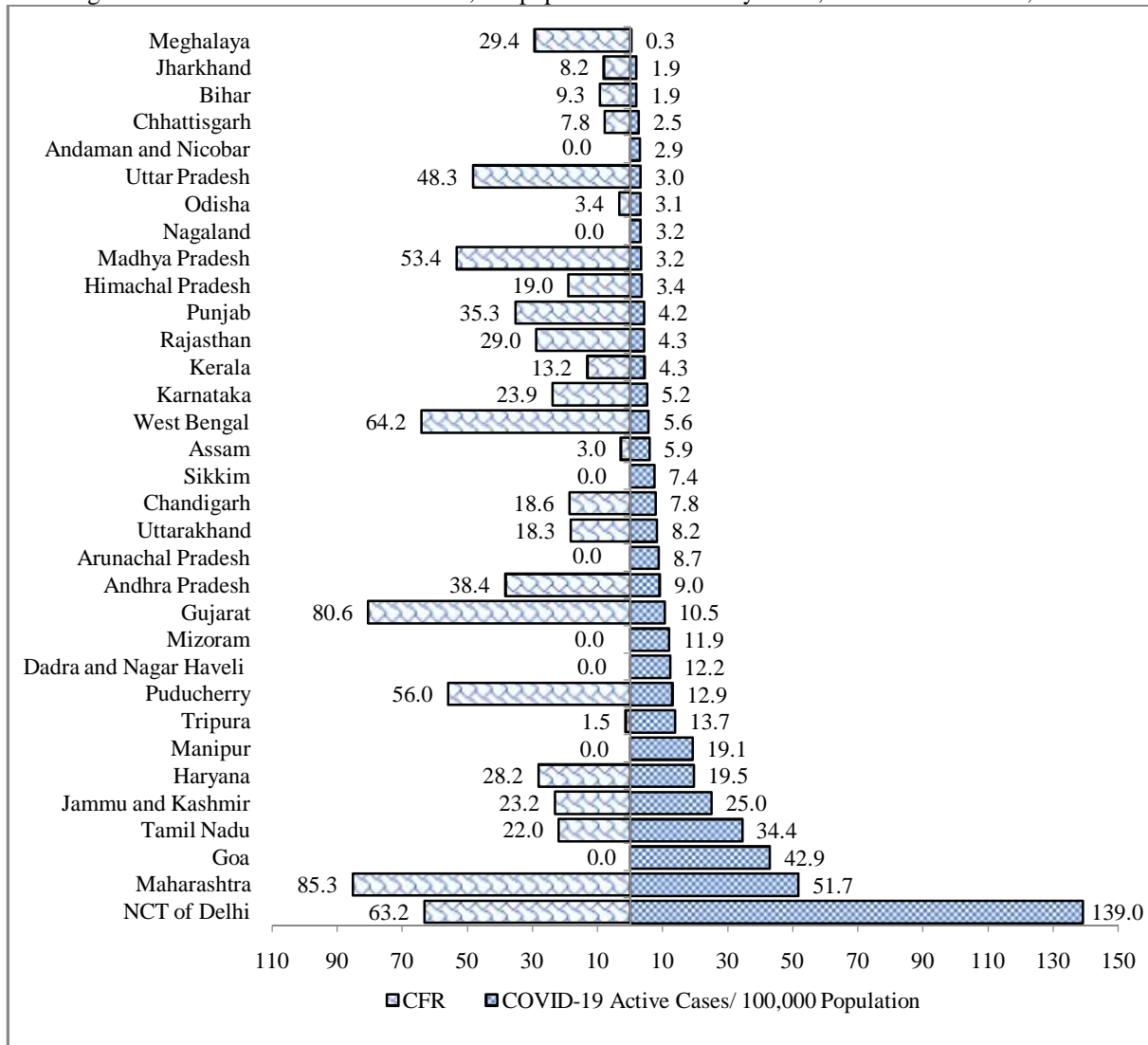
### Results

Data of COVID-19 regarding active cases, cured and deaths has been used to depict the disease pattern across the Indian states in the form of statistical models, and the results as per the best Modified Second-Degree Polynomial are presented. Figure 1 reveals COVID-19 Active Cases and Case Fatality Rates by states of India as on 21 June, 2020 indicating that the worst state with highest number of active cases 139 per 100,000 population was Delhi with Case Fatality Rate (63.2 per 1000 Cured and Deaths) followed by Maharashtra with similar figures 51.6 and 85.3 respectively, and the best state was Meghalaya with active cases 0.3 with alarming Case Fatality Rate 29.4 followed by Jharkhand with respective Rates 1.9 and 8.2.

The Modified Quadratic Model for all state of India fitted well with Coefficient of Determinants ( $R^2=0.842$ ), with varying trends of states, indicating state-wise patterns (Figure 2).

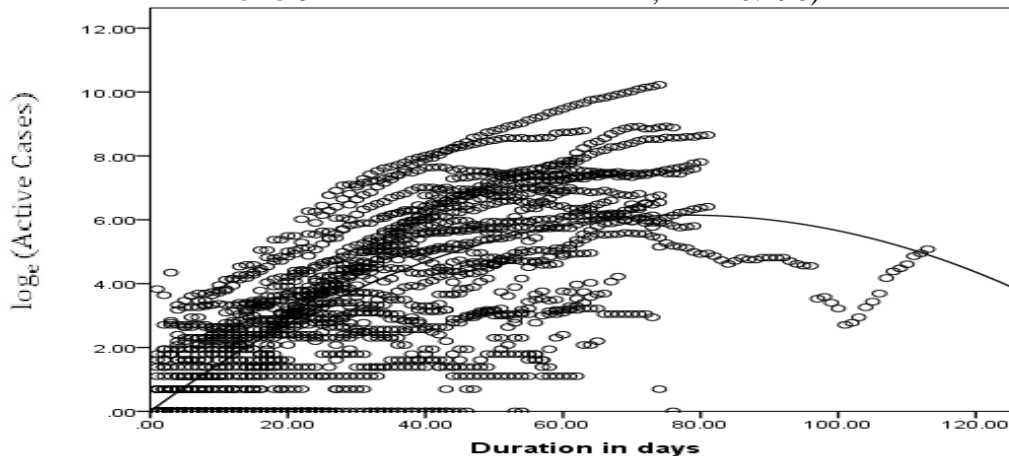
Table 1 reveals COVID-19 Cases and Deaths by states of India indicating that the best state in controlling the Active Case load of COVID-19 was Andaman and Nicobar (Inactive Case Load =100%) followed by Ladakh (98%), Punjab (91%), and worst was Manipur (Inactive Case Load = 8%), followed by Goa (14%), Maharashtra (30%), Odisha (30%), Chandigarh (30%), Assam (31%) and so on.

Figure 1: COVID-19 Active Cases /100,000 population and CFR by states, India as on June 21, 2020



Source: Estimate was obtained from the data extracted from “#IndiaFightsCorona COVID-19 in India, Corona Virus Tracker | mygov.in.” <https://www.mygov.in/covid-19> (accessed Jun. 21, 2020).

Figure 2: Modified Quadratic Model fitted for active cases of states of India (Active Cases =  $1.073 e^{1543.21 \times 10^{-4}t - 97.86 \times 10^{-5}t^2}$ ;  $R^2 = 0.496$ )



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**Table 1: COVID-19 Cases and Deaths by states of India, May 1, 2020**

Day	Cured	Deaths	Active Cases (AC)	Case Load	Inactive Case Load	Estimated Active Cases (EAC)	Active Case Ratio(95% CI)
Maharashtra: $R^2 = 0.999$ , $F = 13737$ , $p < 0.001$ , $b_0 = 20957.42 \times 10^{-4}$ , $b_1 = 1838.02 \times 10^{-4}$ , $b_2 = -10.07 \times 10^{-4}$							
14	0	2	65	97.0	3.0	87	
21	25	6	155	83.3	16.7	247	2.83 (2.12, 3.78)
28	42	24	424	86.5	13.5	634	2.56 (2.13, 3.08)
35	208	127	1426	81.0	19.0	1473	2.32 (2.08, 2.59)
42	365	211	3075	84.2	15.8	3098	2.1 (1.98, 2.24)
49	1076	323	6229	81.7	18.3	5904	1.91 (1.83, 1.99)
56	2000	521	9775	79.5	20.5	10194	1.73 (1.67, 1.78)
63	3800	779	15649	77.4	22.6	15947	1.56 (1.53, 1.6)
70	7088	1135	22483	73.2	26.8	22601	1.42 (1.39, 1.45)
77	10318	1390	27589	70.2	29.8	29021	1.28 (1.26, 1.31)
West Bengal: $R^2 = 0.995$ , $F = 4666.5$ , $p < 0.001$ , $b_0 = 15956.26 \times 10^{-4}$ , $b_1 = 1480.99 \times 10^{-4}$ , $b_2 = -8.83 \times 10^{-4}$							
14	0	2	24	92.3	7.7	33	
21	13	3	75	82.4	17.6	75	2.27 (1.43, 3.6)
28	36	7	147	77.4	22.6	156	2.08 (1.58, 2.75)
35	73	12	307	78.3	21.7	298	1.91 (1.57, 2.33)
42	109	20	568	81.5	18.5	523	1.75 (1.53, 2.01)
49	218	133	908	72.1	27.9	840	1.61 (1.45, 1.78)
56	499	190	1374	66.6	33.4	1238	1.47 (1.36, 1.6)
63	1006	244	1575	55.8	44.2	1673	1.35 (1.26, 1.45)
70	1136	253	1714	55.2	44.8	2074	1.24 (1.16, 1.33)
Uttar Pradesh: $R^2 = 0.994$ , $F = 5293.4$ , $p < 0.001$ , $b_0 = -17705.56 \times 10^{-4}$ , $b_1 = 2816.18 \times 10^{-4}$ , $b_2 = -21.15 \times 10^{-4}$							
14	5	0	10	66.7	33.3	6	
21	11	0	22	66.7	33.3	25	4.28 (2.03, 9.03)
28	14	0	87	86.1	13.9	86	3.48 (2.18, 5.55)
35	21	3	281	92.1	7.9	244	2.83 (2.22, 3.59)
42	49	5	603	91.8	8.2	560	2.3 (1.99, 2.65)
49	140	20	1134	87.6	12.4	1045	1.87 (1.69, 2.06)
56	400	31	1612	78.9	21.1	1586	1.52 (1.41, 1.64)
63	944	53	1862	65.1	34.9	1956	1.23 (1.15, 1.32)
70	1758	80	1735	48.6	51.4	1961	1.00 (0.94, 1.07)
77	2783	118	1704	37.0	63.0	1598	0.81 (0.76, 0.87)
84	3066	127	1982	38.3	61.7	1059	0.66 (0.62, 0.71)
Puducherry: $R^2 = 0.994$ , $F = 336.1$ , $p < 0.001$ , $b_0 = 40593.86 \times 10^{-4}$ , $b_1 = -1118.72 \times 10^{-4}$ , $b_2 = 12.74 \times 10^{-4}$							
14	0	0	1	100.0	0.0	16	
21	1	0	4	80.0	20.0	10	0.62 (0.07, 5.59)
28	1	0	6	85.7	14.3	7	0.71 (0.2, 2.51)
35	3	0	4	57.1	42.9	5	0.8 (0.23, 2.84)
42	3	0	5	62.5	37.5	5	0.91 (0.24, 3.38)
49	6	0	3	33.3	66.7	5	1.03 (0.25, 4.31)
56	6	0	6	50.0	50.0	6	1.17 (0.29, 4.66)
63	9	1	8	44.4	55.6	8	1.32 (0.46, 3.81)
70	9	0	9	50.0	50.0	12	1.5 (0.58, 3.88)

Table 1:(Continued)

Day	Cured	Deaths	Active Cases (AC)	Case Load	Inactive Case Load	Estimated Active Cases (EAC)	Active Case Ratio (95% CI)
Gujarat: $R^2 = 0.991$ , $F = 1602$ , $p < 0.001$ , $b_0 = 36079.27 \times 10^{-4}$ , $b_1 = 1591.89 \times 10^{-4}$ , $b_2 = -12.39 \times 10^{-4}$							
14	8	7	72	82.8	17.2	269	
21	25	16	138	77.1	22.9	605	2.25 (1.69, 2.99)
28	64	36	771	88.5	11.5	1205	1.99 (1.66, 2.39)
35	179	103	2125	88.3	11.7	2125	1.76 (1.62, 1.92)
42	527	197	3358	82.3	17.7	3322	1.56 (1.48, 1.65)
49	1500	396	4729	71.4	28.6	4598	1.38 (1.32, 1.45)
56	3562	566	5139	55.5	44.5	5636	1.23 (1.18, 1.28)
63	5219	749	6569	52.4	47.6	6119	1.09 (1.05, 1.13)
Odisha: $R^2 = 0.986$ , $F = 1221.1$ , $p < 0.001$ , $b_0 = 42723.32 \times 10^{-4}$ , $b_1 = -719.31 \times 10^{-4}$ , $b_2 = 16.37 \times 10^{-4}$							
14	0	0	3	100.0	0.0	36	
21	0	0	20	100.0	0.0	33	0.9 (0.27, 3.04)
28	12	1	41	75.9	24.1	35	1.06 (0.62, 1.81)
35	24	1	36	59.0	41.0	43	1.24 (0.8, 1.95)
42	34	1	68	66.0	34.0	63	1.46 (0.98, 2.19)
49	56	1	103	64.4	35.6	108	1.72 (1.26, 2.33)
56	63	2	229	77.9	22.1	217	2.01 (1.6, 2.54)
63	196	3	538	73.0	27.0	513	2.36 (2.03, 2.76)
70	307	6	739	70.2	29.8	1423	2.78 (2.48, 3.1)
Rajasthan: $R^2 = 0.980$ , $F = 1516.5$ , $p < 0.001$ , $b_0 = -22633.14 \times 10^{-4}$ , $b_1 = 2985.67 \times 10^{-4}$ , $b_2 = -22.46 \times 10^{-4}$							
14	3	0	1	25.0	75.0	4	
21	3	0	25	89.3	10.7	20	4.66 (0.63, 34.42)
28	3	0	56	94.9	5.1	76	3.74 (2.34, 6)
35	21	0	253	92.3	7.7	229	3 (2.25, 4.01)
42	21	3	788	97.0	3.0	553	2.41 (2.09, 2.78)
49	183	14	1281	86.7	13.3	1069	1.93 (1.77, 2.11)
56	205	25	1346	85.4	14.6	1659	1.55 (1.44, 1.68)
63	1356	71	1459	50.6	49.4	2066	1.25 (1.16, 1.34)
70	2176	107	1531	40.1	59.9	2064	1 (0.93, 1.07)
77	2992	131	2079	40.0	60.0	1655	0.8 (0.75, 0.86)
84	3404	147	2464	41.0	59.0	1065	0.64 (0.61, 0.68)
Andhra Pradesh: $R^2 = 0.979$ , $F = 866.9$ , $p < 0.001$ , $b_0 = 17715.82 \times 10^{-4}$ , $b_1 = 1788.55 \times 10^{-4}$ , $b_2 = -15.39 \times 10^{-4}$							
14	1	0	8	88.9	11.1	79	
21	1	0	82	98.8	1.2	177	2.25 (1.09, 4.65)
28	5	4	296	97.0	3.0	343	1.93 (1.51, 2.47)
35	16	9	478	95.0	5.0	570	1.66 (1.44, 1.92)
42	120	24	669	82.3	17.7	816	1.43 (1.27, 1.61)
49	287	31	1014	76.1	23.9	1004	1.23 (1.12, 1.36)
56	589	36	1092	63.6	36.4	1063	1.06 (0.97, 1.15)
63	1056	46	988	47.3	52.7	967	0.91 (0.84, 0.99)
70	1621	52	859	33.9	66.1	757	0.78 (0.71, 0.86)
77	1640	53	909	34.9	65.1	509	0.67 (0.61, 0.74)

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Table 1:(Continued)

Day	Cured	Deaths	Active Cases (AC)	Case Load	Inactive Case Load	Estimated Active Cases (EAC)	Active Case Ratio (95% CI)
Delhi: $R^2 = 0.974$ , $F = 1184.5$ , $p < 0.001$ , $b_0 = -25606.44 \times 10^{-4}$ , $b_1 = 3049.25 \times 10^{-4}$ , $b_2 = -21.11 \times 10^{-4}$							
14	2	1	4	57.1	42.9	4	
21	5	1	23	79.3	20.7	18	5.04 (1.74, 14.57)
28	6	2	41	83.7	16.3	75	4.1 (2.46, 6.83)
35	18	7	478	95.0	5.0	251	3.33 (2.42, 4.58)
42	25	19	1025	95.9	4.1	680	2.71 (2.43, 3.02)
49	72	43	1778	93.9	6.1	1499	2.2 (2.04, 2.38)
56	869	54	1702	64.8	35.2	2685	1.79 (1.68, 1.91)
63	1256	64	2802	68.0	32.0	3911	1.46 (1.37, 1.55)
70	2020	73	4449	68.0	32.0	4633	1.18 (1.13, 1.24)
77	3926	129	5278	56.6	43.4	4462	0.96 (0.93, 1)
84	5192	176	5720	51.6	48.4	3494	0.78 (0.75, 0.81)
Manipur: $R^2 = 0.961$ , $F = 24.4$ , $p < 0.05$ , $b_0 = 13527.85 \times 10^{-4}$ , $b_1 = -103.37 \times 10^{-4}$ , $b_2 = 18.85 \times 10^{-4}$							
14	0	0	2	100.0	0.0	1	
21	1	0	1	50.0	50.0	1	0.77 (0.07, 8.49)
28	2	0	0	0.0	100.0	1	-
35	2	0	0	0.0	100.0	1	-
42	2	0	0	0.0	100.0	1	-
49	2	0	0	0.0	100.0	2	-
56	2	0	5	71.4	28.6	4	-
63	2	0	23	92.0	8.0	10	2.33 (0.89, 6.14)
Goa: $R^2 = 0.955$ , $F = 115.6$ , $p < 0.001$ , $b_0 = 79634.49 \times 10^{-4}$ , $b_1 = -5166.25 \times 10^{-4}$ , $b_2 = 78.21 \times 10^{-4}$							
14	0	0	7	100.0	0.0	10	
21	5	0	2	28.6	71.4	2	0.18 (0.04, 0.88)
28	7	0	0	0.0	100.0	1	-
35	7	0	0	0.0	100.0	1	-
42	7	0	0	0.0	100.0	1	-
49	7	0	0	0.0	100.0	4	-
56	7	0	39	84.8	15.2	35	-
63	7	0	43	86.0	14.0	636	18.14 (11.76, 27.98)
Assam: $R^2 = 0.951$ , $F = 175$ , $p < 0.001$ , $b_0 = 28903.59 \times 10^{-4}$ , $b_1 = -1107.13 \times 10^{-4}$ , $b_2 = 28.60 \times 10^{-4}$							
14	0	1	30	96.8	3.2	7	
21	19	1	15	42.9	57.1	6	0.93 (0.5, 1.73)
28	27	1	10	26.3	73.7	8	1.23 (0.55, 2.74)
35	32	1	10	23.3	76.7	12	1.63 (0.68, 3.91)
42	34	2	29	44.6	55.4	27	2.15 (1.05, 4.42)
49	41	2	64	59.8	40.2	76	2.85 (1.84, 4.42)
56	48	4	118	69.4	30.6	287	3.77 (2.78, 5.11)
Telangana: $R^2 = 0.949$ , $F = 583.4$ , $p < 0.001$ , $b_0 = -20905.79 \times 10^{-4}$ , $b_1 = 2918.98 \times 10^{-4}$ , $b_2 = -24.34 \times 10^{-4}$							
14	1	0	2	66.7	33.3	5	
21	1	0	21	95.5	4.5	19	4.25 (1, 18.13)
28	1	1	64	97.0	3.0	65	3.35 (2.05, 5.48)
35	32	7	230	85.5	14.5	171	2.64 (2, 3.48)
42	43	9	452	89.7	10.3	356	2.08 (1.77, 2.44)
49	186	18	640	75.8	24.2	583	1.64 (1.45, 1.85)
56	280	26	685	69.1	30.9	752	1.29 (1.16, 1.44)
63	458	28	577	54.3	45.7	763	1.02 (0.91, 1.13)
70	750	30	383	32.9	67.1	611	0.8 (0.7, 0.91)
77	971	34	504	33.4	66.6	385	0.63 (0.55, 0.72)
84	1015	40	606	36.5	63.5	191	0.5 (0.44, 0.56)

Table 1: (Continued)

Day	Cured	Deaths	Active Cases (AC)	Case Load	Inactive Case Load	Estimated Active Cases (EAC)	Active Case Ratio (95% CI)
Karnataka: $R^2 = 0.941$ , $F = 448.9$ , $p < 0.001$ , $b_0 = 24914.93 \times 10^{-4}$ , $b_1 = 0943.84 \times 10^{-4}$ , $b_2 = -5.84 \times 10^{-4}$							
14	2	1	23	88.5	11.5	40	
21	5	3	68	89.5	10.5	68	1.68 (1.05, 2.69)
28	12	4	128	88.9	11.1	107	1.58 (1.18, 2.13)
35	37	6	183	81.0	19.0	161	1.5 (1.19, 1.88)
42	104	14	266	69.3	30.7	227	1.41 (1.17, 1.71)
49	177	18	306	61.1	38.9	303	1.34 (1.13, 1.57)
56	282	25	299	49.3	50.7	383	1.26 (1.08, 1.48)
63	386	30	378	47.6	52.4	456	1.19 (1.02, 1.39)
70	496	36	560	51.3	48.7	512	1.12 (0.99, 1.28)
77	556	41	865	59.2	40.8	544	1.06 (0.95, 1.18)
Jammu and Kashmir: $R^2 = 0.935$ , $F = 402.4$ , $p < 0.001$ , $b_0 = -8850.11 \times 10^{-4}$ , $b_1 = 2371.60 \times 10^{-4}$ , $b_2 = -19.53 \times 10^{-4}$							
14	0	0	4	100.0	0.0	8	
21	1	2	28	90.3	9.7	25	3.26 (1.14, 9.29)
28	4	2	100	94.3	5.7	68	2.69 (1.77, 4.09)
35	6	4	214	95.5	4.5	152	2.22 (1.75, 2.82)
42	51	5	285	83.6	16.4	279	1.84 (1.54, 2.19)
49	112	6	376	76.1	23.9	423	1.52 (1.3, 1.77)
56	254	8	404	60.7	39.3	529	1.25 (1.09, 1.44)
63	368	9	459	54.9	45.1	547	1.03 (0.9, 1.18)
70	542	12	567	50.6	49.4	467	0.85 (0.75, 0.97)
77	678	18	694	49.9	50.1	329	0.71 (0.63, 0.79)
Madhya Pradesh: $R^2 = 0.929$ , $F = 286.2$ , $p < 0.001$ , $b_0 = 21461.59 \times 10^{-4}$ , $b_1 = 2249.36 \times 10^{-4}$ , $b_2 = -22.48 \times 10^{-4}$							
14	0	6	98	94.2	5.8	128	
21	0	16	243	93.8	6.2	357	2.78 (2.2, 3.52)
28	65	57	1186	90.7	9.3	798	2.23 (1.95, 2.56)
35	203	83	1566	84.6	15.4	1430	1.79 (1.66, 1.93)
42	482	137	2100	77.2	22.8	2055	1.44 (1.35, 1.53)
49	1231	193	1828	56.2	43.8	2370	1.15 (1.08, 1.23)
56	2171	237	2018	45.6	54.4	2193	0.93 (0.87, 0.99)
63	2733	267	2735	47.7	52.3	1628	0.74 (0.7, 0.79)
Chandigarh: $R^2 = 0.919$ , $F = 174.8$ , $p < 0.001$ , $b_0 = -60806.47 \times 10^{-4}$ , $b_1 = 3362.61 \times 10^{-4}$ , $b_2 = -25.13 \times 10^{-4}$							
14	0	0	16	100.0	0.0	0	
21	7	0	11	61.1	38.9	1	5.69 (2.64, 12.26)
28	7	0	14	66.7	33.3	4	4.45 (2.02, 9.79)
35	14	0	13	48.1	51.9	14	3.48 (1.63, 7.39)
42	17	0	39	69.6	30.4	37	2.72 (1.45, 5.09)
49	21	1	89	80.2	19.8	79	2.12 (1.46, 3.09)
56	28	3	156	83.4	16.6	130	1.66 (1.28, 2.15)
63	57	3	140	70.0	30.0	169	1.3 (1.03, 1.63)
70	57	3	142	70.3	29.7	172	1.01 (0.8, 1.28)
Tripura: $R^2 = 0.912$ , $F = 82.6$ , $p < 0.001$ , $b_0 = -385094.43 \times 10^{-4}$ , $b_1 = 23195.92 \times 10^{-4}$ , $b_2 = -308.15 \times 10^{-4}$							
14	1	0	1	50.0	50.0	0	
21	2	0	0	0.0	100.0	0	-
28	2	0	14	87.5	12.5	10	-
35	2	0	148	98.7	1.3	138	14.12 (8.16, 24.43)
42	85	0	82	49.1	50.9	95	0.69 (0.53, 0.9)
49	133	0	40	23.1	76.9	3	0.03 (0.02, 0.05)



**Dynamics of COVID-19 Cases in India: A Statistical Model**

Table 1:(Continued)

Day	Cured	Deaths	Active Cases (AC)	Case Load	Inactive Case Load	Estimated Active Cases (EAC)	Active Case Ratio (95% CI)
Tamil Nadu: $R^2 = 0.883$ , $F = 218$ , $p < 0.001$ , $b_0 = -4608.70 \times 10^{-4}$ , $b_1 = 2354.21 \times 10^{-4}$ , $b_2 = -15.41 \times 10^{-4}$							
14	1	0	2	66.7	33.3	13	
21	1	1	27	93.1	6.9	45	3.56 (0.85, 14.98)
28	6	1	302	97.7	2.3	137	3.06 (2.07, 4.54)
35	21	8	805	96.5	3.5	362	2.63 (2.31, 3.01)
42	180	15	1072	84.6	15.4	820	2.26 (2.07, 2.48)
49	752	20	911	54.1	45.9	1596	1.95 (1.78, 2.13)
56	1258	27	1038	44.7	55.3	2673	1.67 (1.53, 1.83)
63	1547	37	3825	70.7	29.3	3849	1.44 (1.34, 1.54)
70	2240	66	7368	76.2	23.8	4765	1.24 (1.19, 1.29)
77	5882	87	7222	54.7	45.3	5073	1.06 (1.03, 1.1)
Haryana: $R^2 = 0.863$ , $F = 191.4$ , $p < 0.001$ , $b_0 = 8629.78 \times 10^{-4}$ , $b_1 = 1114.94 \times 10^{-4}$ , $b_2 = -6.04 \times 10^{-4}$							
14	0	0	15	100.0	0.0	10	
21	11	0	17	60.7	39.3	19	1.88 (0.94, 3.77)
28	21	0	19	47.5	52.5	33	1.77 (0.92, 3.41)
35	25	1	64	71.1	28.9	56	1.67 (1, 2.79)
42	34	3	162	81.4	18.6	88	1.58 (1.18, 2.11)
49	127	3	124	48.8	51.2	131	1.49 (1.18, 1.88)
56	183	3	110	37.2	62.8	184	1.4 (1.08, 1.81)
63	254	6	257	49.7	50.3	242	1.32 (1.06, 1.65)
70	337	11	382	52.3	47.7	301	1.24 (1.06, 1.46)
77	598	14	316	34.1	65.9	353	1.17 (1.01, 1.36)
84	648	14	331	33.3	66.7	391	1.11 (0.95, 1.29)
Punjab: $R^2 = 0.838$ , $F = 144.4$ , $p < 0.001$ , $b_0 = 10689.94 \times 10^{-4}$ , $b_1 = 1295.61 \times 10^{-4}$ , $b_2 = -6.74 \times 10^{-4}$							
14	0	1	20	95.2	4.8	16	
21	1	1	36	94.7	5.3	33	2.1 (1.22, 3.63)
28	1	5	51	89.5	10.5	65	1.97 (1.28, 3.01)
35	5	11	135	89.4	10.6	119	1.84 (1.33, 2.54)
42	31	16	172	78.5	21.5	205	1.72 (1.38, 2.16)
49	67	17	214	71.8	28.2	330	1.61 (1.32, 1.97)
56	112	20	640	82.9	17.1	499	1.51 (1.29, 1.76)
63	157	31	1574	89.3	10.7	705	1.41 (1.29, 1.55)
70	1257	32	657	33.8	66.2	932	1.32 (1.21, 1.45)
77	1794	38	173	8.6	91.4	1154	1.24 (1.05, 1.46)
Jharkhand: $R^2 = 0.836$ , $F = 83.8$ , $p < 0.001$ , $b_0 = 8933.61 \times 10^{-4}$ , $b_1 = 1821.36 \times 10^{-4}$ , $b_2 = -22.50 \times 10^{-4}$							
14	0	2	22	91.7	8.3	20	
21	0	2	44	95.7	4.3	42	2.06 (1.24, 3.44)
28	19	3	83	79.0	21.0	69	1.65 (1.15, 2.38)
35	33	3	89	71.2	28.8	91	1.33 (0.98, 1.79)
42	79	3	90	52.3	47.7	97	1.06 (0.79, 1.43)
49	127	3	101	43.7	56.3	83	0.85 (0.64, 1.13)
56	127	3	101	43.7	56.3	57	0.68 (0.52, 0.9)

Table 1: (Continued)

Day	Cured	Deaths	Active Cases (AC)	Case Load	Inactive Case Load	Estimated Active Cases (EAC)	Active Case Ratio (95% CI)
Bihar: $R^2 = 0.819$ , $F = 63.4$ , $p < 0.001$ , $b_0 = 7819.77 \times 10^{-4}$ , $b_1 = 1552.59 \times 10^{-4}$ , $b_2 = -9.62 \times 10^{-4}$							
14	0	1	29	96.7	3.3	16	
21	0	1	59	98.3	1.7	37	2.34 (1.5, 3.65)
28	37	2	46	54.1	45.9	79	2.13 (1.45, 3.13)
35	46	2	180	78.9	21.1	154	1.94 (1.4, 2.68)
42	98	3	370	78.6	21.4	272	1.76 (1.48, 2.11)
49	297	5	269	47.1	52.9	437	1.61 (1.37, 1.88)
56	438	7	573	56.3	43.7	638	1.46 (1.26, 1.69)
63	571	10	1093	65.3	34.7	849	1.33 (1.2, 1.47)
Kerala: $R^2 = 0.799$ , $F = 179.1$ , $p < 0.001$ , $b_0 = -29669.03 \times 10^{-4}$ , $b_1 = 1975.67 \times 10^{-4}$ , $b_2 = -12.37 \times 10^{-4}$							
14	0	0	3	100.0	0.0	1	
21	0	0	3	100.0	0.0	2	2.94 (0.59, 14.59)
28	0	0	3	100.0	0.0	5	2.61 (0.53, 12.92)
35	3	0	0	0.0	100.0	11	-
42	3	0	14	82.4	17.6	23	-
49	3	0	24	88.9	11.1	42	1.81 (0.94, 3.5)
56	4	0	105	96.3	3.7	68	1.61 (1.03, 2.5)
63	23	2	216	89.6	10.4	97	1.42 (1.13, 1.8)
70	70	2	264	78.6	21.4	122	1.26 (1.05, 1.51)
77	211	3	173	44.7	55.3	136	1.12 (0.92, 1.35)
84	323	3	101	23.7	76.3	134	0.99 (0.77, 1.26)
91	359	4	123	25.3	74.7	118	0.88 (0.67, 1.14)
98	462	4	36	7.2	92.8	91	0.78 (0.54, 1.13)
105	489	4	31	5.9	94.1	63	0.69 (0.43, 1.11)
112	497	4	141	22.0	78.0	38	0.61 (0.41, 0.9)
119	502	4	160	24.0	76.0	21	0.54 (0.43, 0.68)
Himachal Pradesh: $R^2 = 0.656$ , $F = 27.6$ , $p < 0.001$ , $b_0 = 184205.24 \times 10^{-4}$ , $b_1 = -7274.61 \times 10^{-4}$ , $b_2 = 80.64 \times 10^{-4}$							
14	1	1	4	66.7	33.3	18336	
21	6	1	21	75.0	25.0	812	0.04 (0.02, 0.13)
28	16	1	18	51.4	48.6	79	0.1 (0.05, 0.18)
35	18	1	21	52.5	47.5	17	0.22 (0.11, 0.4)
42	28	1	11	27.5	72.5	8	0.47 (0.23, 0.98)
49	38	2	6	13.0	87.0	8	1.05 (0.39, 2.83)
56	39	2	33	44.6	55.4	20	2.3 (0.97, 5.5)
63	54	3	53	48.2	51.8	99	5.08 (3.29, 7.84)
Andaman and Nicobar Islands: $R^2 = 0.501$ , $F = 3$ , $p < 0.001$ , $b_0 = -95967.69 \times 10^{-4}$ , $b_1 = 7441.67 \times 10^{-4}$ , $b_2 = -109.86 \times 10^{-4}$							
14	0	0	10	100.0	0.0	0	
21	10	0	1	9.1	90.9	3	12.4 (1.59, 96.87)
28	11	0	6	35.3	64.7	14	4.23 (0.51, 35.1)
35	15	0	18	54.5	45.5	20	1.44 (0.57, 3.63)
42	32	0	1	3.0	97.0	10	0.49 (0.07, 3.68)
49	33	0	0	0.0	100.0	2	-
56	33	0	0	0.0	100.0	0	-
63	33	0	0	0.0	100.0	0	-

Table 1: (Continued)

Day	Cured	Deaths	Active Cases (AC)	Case Load	Inactive Case Load	Estimated Active Cases (EAC)	Active Case Ratio (95% CI)
Ladakh: $R^2 = 0.416$ , $F = 20$ , $p < 0.001$ , $b_0 = 41789.97 \times 10^{-4}$ , $b_1 = -1214.88 \times 10^{-4}$ , $b_2 = 14.96 \times 10^{-4}$							
14	0	0	10	100.0	0.0	16	
21	3	0	10	76.9	23.1	10	0.62 (0.26, 1.48)
28	3	0	11	78.6	21.4	7	0.71 (0.3, 1.68)
35	10	0	5	33.3	66.7	6	0.83 (0.29, 2.38)
42	14	0	4	22.2	77.8	6	0.96 (0.26, 3.56)
49	14	0	4	22.2	77.8	6	1.11 (0.28, 4.43)
56	16	0	6	27.3	72.7	8	1.28 (0.36, 4.55)
63	17	0	25	59.5	40.5	12	1.49 (0.61, 3.62)
70	22	0	21	48.8	51.2	20	1.72 (0.96, 3.07)
77	43	0	1	2.3	97.7	40	1.99 (0.27, 14.8)
Uttarakhand: $R^2 = 0.327$ , $F = 12.1$ , $p < 0.001$ , $b_0 = 14135.35 \times 10^{-4}$ , $b_1 = 685.31 \times 10^{-4}$ , $b_2 = -6.01 \times 10^{-4}$							
14	0	0	5	100.0	0.0	10	
21	2	0	14	87.5	12.5	13	1.39 (0.5, 3.87)
28	5	0	30	85.7	14.3	17	1.31 (0.7, 2.48)
35	9	0	33	78.6	21.4	22	1.24 (0.76, 2.03)
42	25	0	23	47.9	52.1	25	1.17 (0.69, 1.99)
49	36	0	22	37.9	62.1	28	1.1 (0.61, 1.98)
56	46	1	16	25.4	74.6	29	1.04 (0.55, 1.98)
63	51	1	30	36.6	63.4	28	0.98 (0.53, 1.8)
70	53	1	68	55.7	44.3	26	0.92 (0.6, 1.42)
Chhattisgarh: $R^2 = 0.278$ , $F = 6$ , $p < 0.01$ , $b_0 = 77183.54 \times 10^{-4}$ , $b_1 = -2704.53 \times 10^{-4}$ , $b_2 = 31.71 \times 10^{-4}$							
14	2	0	7	77.8	22.2	95	
21	9	0	1	10.0	90.0	31	0.33 (0.04, 2.66)
28	13	0	20	60.6	39.4	14	0.45 (0.06, 3.33)
35	26	0	10	27.8	72.2	8	0.61 (0.29, 1.3)
42	34	0	4	10.5	89.5	7	0.83 (0.26, 2.65)
49	36	0	23	39.0	61.0	8	1.13 (0.39, 3.28)
56	54	0	5	8.5	91.5	12	1.55 (0.59, 4.07)
63	59	0	42	41.6	58.4	26	2.11 (0.84, 5.34)
70	59	0	56	48.7	51.3	75	2.88 (1.93, 4.3)

The trends in performance of the states of India in controlling of the COVID-19 has been analysed using Modified Second Degree Polynomial Model, so that unusual in trends of cure of COVID-19 are traced and reasoned out. The model is fitted well for state Maharashtra with Coefficient of Determinant ( $R^2=0.999$ ) followed by states of West Bengal ( $R^2=0.995$ ), Uttar Pradesh ( $R^2=0.994$ ), Puducherry ( $R^2=0.994$ ), Gujarat ( $R^2=0.991$ ), Odisha ( $R^2=0.986$ ), Rajasthan ( $R^2=0.980$ ), etc., and worst for the state of Chhattisgarh ( $R^2=0.278$ ), Uttarakhand ( $R^2=0.372$ ), Ladakh ( $R^2=0.416$ ), and Andaman and Nicobar Island ( $R^2=0.501$ ).

In Maharashtra, Active Case Ratio from 14 to 21 days was 2.83 with 95% Confidence Interval (CI) 2.12-3.78 and decreased to 1.28 (95% CI: 1.26 - 1.31) by 77 days. Inactive Case Load was increased by around 14% from 14th days to 21st days. After 42 days, it again starts to steadily increased, and was continued by 77 days. Similar pattern was also observed in the state of West Bengal, Uttar Pradesh, Puducherry, Gujarat, Rajasthan, Andhra Pradesh, Telangana, Karnataka and so on with higher Inactive Case Load than Maharashtra. In addition, increased Active Case Ratio was observed in the state of Chhattisgarh, Ladakh, Himachal Pradesh, Assam, Odisha, and Puducherry.

Figure 3a: Average Active Cases ( $R^2 = 0.995$ ) and Active Case Ratio with 95% CI in West Bengal

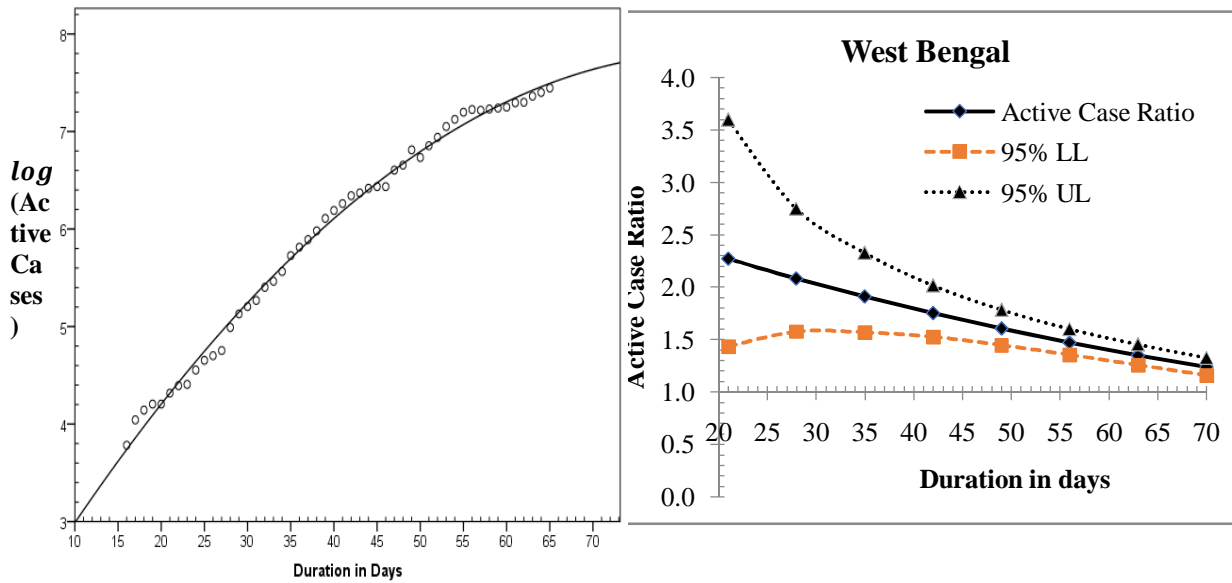


Figure 3b: Average Active Cases ( $R^2 = 0.999$ ) and Active Case Ratio with 95% CI in Maharashtra

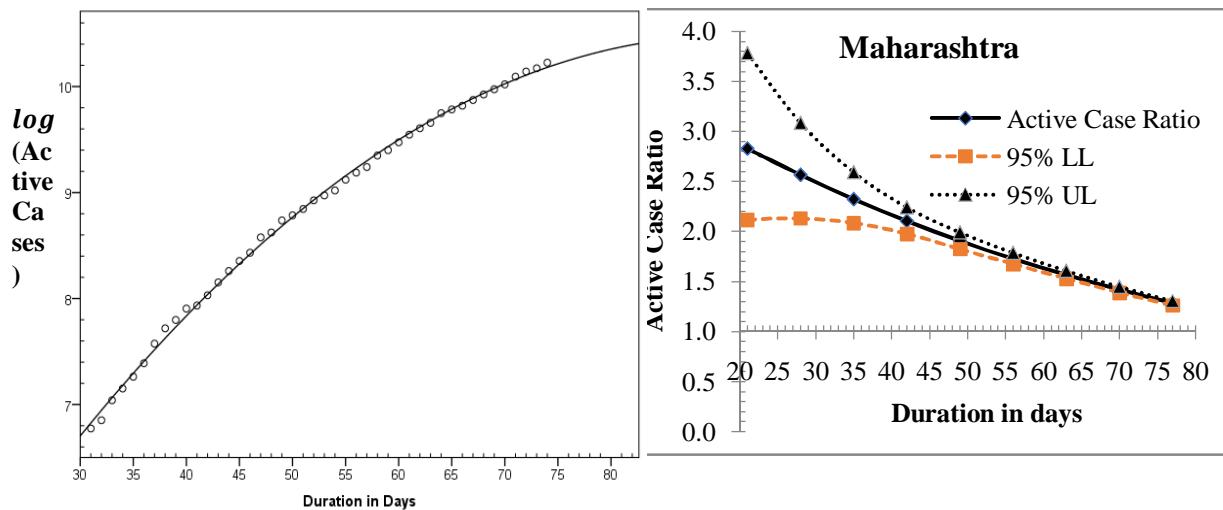
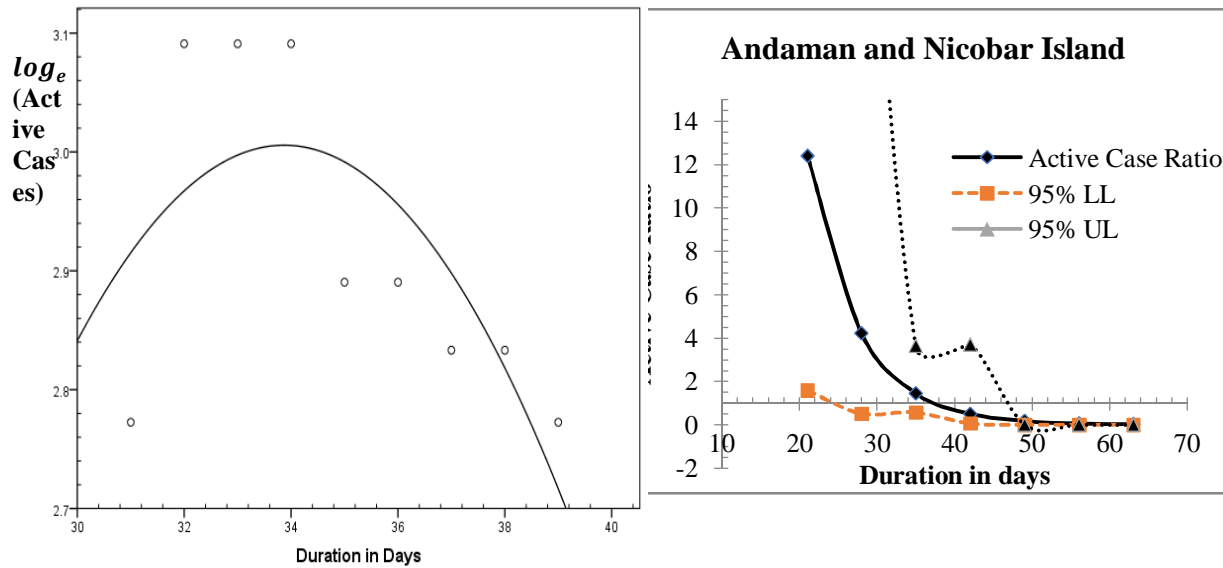


Figure 3a, reveals Average Active Cases and Active Case Ratio with 95% CI in West Bengal, indicating that the COVID-19 Active cases increased consistently from 15<sup>th</sup> days to 65<sup>th</sup> days. Furthermore, the observed Active Cases were fitted well with Modified Second-Degree Polynomial Model ( $R^2 = 0.496$ ). Fig 3a also indicated that rate of increment in active cases were slow and near to stabilization at 70<sup>th</sup> days, i.e. the point where Active Cases was same as that of the cases at just previous days. Similarly, the observed Active Cases of Maharashtra state fitted well with Modified Second-Degree Polynomial Model ( $R^2 = 0.999$ ) as shown in fig. 3b. The Active Cases exhibited increasing trend, and the rate of increment in active cases was similar as in West Bengal (Figure 3a).

However, Active cases in Andaman and Nicobar Island went down near to X-Axis, indicating of line of stabilization point at nearly 38<sup>th</sup> days i.e. Active Cases of COVID-19 were almost not there (Fig 3c). In other words, Cases Load decreased fast from 14<sup>th</sup> day to 20<sup>th</sup> days and thereafter, increased up to 35<sup>th</sup> day and then decreased around 50% by 42<sup>th</sup> days. Thereafter, state achieved 100% Inactive Case Load at 49<sup>th</sup> days and remained constant till 65<sup>th</sup> days.

Figure 3c: Average Active Cases ( $R^2 = 0.501$ ) and Active Case Ratio with 95% CI in Andaman and Nicobar



**Discussion**

The COVID-2019 as a pandemic was declared by WHO on March 11, 2020, killed around 452,669 humans in the world by June 18, 2020 (Coronavirus Update, 18 June, 2020). COVID-19 cases and deaths increased exponentially in the world. Medical professionals, Researchers and Scientists are working hard in controlling the disease from day one. A large number of studies have been published in national and international journals by experts to examine the characteristics of COVID-19 infectious disease (Prasad et al., 2020; Dwivedi et al., 2020; Bhattacharyya et al., 2020). Moreover, there has been history of research on epidemic and pandemic deceases in statistical physics (Pastor et al., 2015; Wang et al., 2016). Furthermore, exponential, quadratic and cubic statistical models have been used to estimate the trends, and to predict the number of cases (Rasad et al., 2020; Dwivedi et al., 2020; Bhattacharyya et al., 2020). However, in the present study Modified Quadratic Model has been used keeping in mind nature of model and disease characteristic.

Modified Quadratic Model for all state together fitted well with Coefficient of Determinant ( $R^2=0.842$ ), with similar patterns and with varying degree of states. Whereas, Bhattacharyya (2020) have used four degree polynomial on number of new cases and indicated the similar coefficient of determinants (Bhattacharyya et al., 2020), not much useful to make the necessary action as lockdown, de-lockdown and standardization of line of treatment to enhance health and economic for the benefits of the community, as the shape of the model cyclic and needed to estimate reoccurrence of the disease. According to the present study, the best state in controlling the Case Load of COVID-19 was Andaman and Nicobar (Inactive Case Load=100%) followed by Ladakh, Punjab, etc. and the worst was Manipur (8%), followed by Goa, Maharashtra, Odisha, Chandigarh, Assam and so on.

The levels and trends of performance of the states of India in controlling of the COVID-19 has been analysed using Modified Second Degree Polynomial, so that unusual trends in cure of COVID-19 are traced and reasoned out. For Maharashtra, model is fitted well with  $R^2= 0.999$ . Furthermore, Modified Second Degree Polynomial Model is fitted well for the states of West Bengal, Uttar Pradesh, Puducherry, Gujarat, Odisha, Rajasthan, etc. and

worst for the state of Chhattisgarh ( $R^2=0.278$ ) followed by Uttarakhand, Ladakh, and Andaman and Nicobar Island.

In Maharashtra, Active Case Ratio on 21<sup>st</sup> day was 2.83 as compared to at 14<sup>th</sup> day, decreased to 1.28 by 77<sup>th</sup> day. Inactive Case Load was increased steadily from 14<sup>th</sup> to 77<sup>th</sup> days. Inactive Case Load observed higher in the state of West Bengal, Uttar Pradesh, Puducherry, Gujarat, Rajasthan, Andhra Pradesh, Telangana, Karnataka and so on than Maharashtra, but active cases did not reach at declining point. In addition, increasing pattern in Active Case Ratio was observed in the state of Chhattisgarh, Ladakh, Himachal Pradesh, Assam, Odisha, and Puducherry indicating either state have inadequate Health Services or due to movement of labour or due to both. However, Active cases in Andaman and Nicobar Island went down near line of stabilization point at nearly 38<sup>th</sup> days i.e. Active Cases of COVID-19 is almost finished. Andaman and Nicobar state has achieved 100% Inactive Case Load at 49<sup>th</sup> day and remained constant up to 65<sup>th</sup> day. Furthermore, fluctuation in Inactive Case Load of Maharashtra, Delhi, etc., could not be expressed by the model due to movement of labour and density of population.

## Conclusion

The COVID-19 Active Case Rate varied from 0 to 139 per 100,000 populations with Delhi as highest followed by Maharashtra. The best state was Meghalaya, and Andaman and Nicobar with respective rates 0.3 and 2.9. The reason for wide variation in active cases of COVID-19 may be unrest in community due to labour movement. Hence, the disease COVID-19 could have been easily controlled by restricting public movement, and providing them necessities. The Modified Quadratic Model has been the best fit and can be used to assess the performance of the states in controlling the COVID-19. The model fitted well with Coefficient of Determinant ( $R^2$ ) as highest ( $R^2=0.999$ ), with more than 0.80  $R^2$  in 23 states, furthermore, the state with  $R^2$  less than 0.80 had smaller number of cases of COVID-19, hence, model did not fit well.

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## Dynamics of COVID-19 Cases in India: A Statistical Model

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